Inventory and marketing policy in a supply chain of a perishable product

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ABSTRACT

We investigate a two-echelon supply chain comprising a manufacturer and a retailer, who are engaged in a Stackelberg game in which they set the terms of a wholesale price contract for a perishable product. Product demand depends on the selling price, the investment in advertising, and the time a unit spends on the shelf before being sold. The investment in advertising can be made either by the manufacturer, by the retailer or in a cooperative manner. The parties apply an economic order quantity policy, where the cycle length is set endogenously by the leader of the supply chain. We model the decisions of the parties regarding pricing, advertising investment and cycle length, and we investigate how different power balances between the parties affect their decisions and other supply chain measures at equilibrium. In particular, we analyze two cases: manufacturer-leader and retailer-leader. For each one, equilibrium is obtained for two demand forms (one in which the effects of price and advertising on demand are additive, and one in which they are multiplicative). We find that for a given type of advertising investment (cooperative/non-cooperative) and a given cycle length, the variable profit of each party is determined only by that party’s role in the game (leader/follower) and not by its identity (manufacturer/retailer). This result is valid for general formulations of the advertising cost function and of the demand. Interestingly, the type of advertising investment (cooperative/non-cooperative) depends on the sequence of decisions, where at equilibrium, the participation of each party in the advertising investment is determined only by its channel power.

1. Introduction

Management of supply chains involving perishable products—broadly defined as products with a short lifetime or that deteriorate over time (e.g., fresh foods, agricultural products, pharmaceuticals and medications, among others)—is a notoriously difficult problem with many specific challenges (e.g., Kouki et al., 2013; Kouki et al., 2015; Kouki et al., 2018; Haijema and Minner, 2016). These challenges arise from the fact that perishable products lose their quality and value over a specified time period even when handled correctly throughout the supply chain. Yet, the need for effective policies to manage supply chains of perishable products is becoming increasingly salient, as the standard of living is rising and consumers are increasingly seeking out high-quality fresh foods, as opposed to canned, frozen or otherwise non-perishable goods. For example, according to Li and Teng (2018), the sales mix in the US grocery industry comprises 50% perishable foods, 30% nonperishable foods and 20% nonfood items. Moreover, fresh food accounts for up to 40 percent of grocers’ revenue and one-third of the cost of goods sold (Glatzel et al., 2016).

When devising supply chain management policies for perishable products, it is necessary to take into account the direct and indirect effects of product perishability on inventory level (see Nahmias, 1982, 2011; Karaesmen et al., 2011; and Bakker et al., 2012 for extensive surveys on perishable-inventory management). The direct effect results from spoilage; i.e., after a given period of time, unsold perishable products become unusable and disappear from inventory. The indirect effect results from the fact that demand for a specific inventory of perishable products decreases over time, owing to consumers’ preference for fresher items (see e.g., Herbon, 2017). As for the indirect effect, Sarkar et al. (1997), for example, claimed that consumers tend to feel less confident purchasing perishable products whose expiration dates are approaching. Tsairos and Heilman (2005) observed that customers frequently (more than 50%) check the expiration date before making a purchase, and that willingness to purchase a perishable product decreases throughout its shelf life.

This work deals with the indirect effect of perishability on inventory, by analyzing a Stackelberg game in which two members of a supply chain—a manufacturer and a retailer—jointly set the terms of a wholesale price contract for a perishable product. Assuming an economic order quantity (EOQ) framework, we consider multiple scenarios that differ in the identity of the party who leads the game and in the decision rights allocated to each party with respect to advertising.
investment and replenishment policy. We further assume that demand for the product is affected by the product’s price, the supply chain members’ investments in advertising, and the product’s age. The managerial contribution of this work is in addressing the following research questions:

- Which party should be the leader of the supply chain in order to achieve coordination?
- Which sequence of decisions leads to cooperative advertising (i.e., joint investment in advertising as opposed to a single party bearing the entire cost; see below)?
- Do customers and the parties benefit from cooperative advertising?
- Are the parties’ strategies affected by the demand form?
- How does the product’s perishability affect equilibrium?

The theoretical contribution of this paper is in obtaining results that are valid for any advertising cost function and for any demand form. In particular, we find that for a given type of advertising investment (cooperative/non-cooperative), at conditional equilibrium (i.e., when the length of the replenishment period is assumed to be given): (i) the net unit profit margin and the variable profit per cycle of each party are determined only by the party’s role in the game (leader/follower) and not by the party’s identity (manufacturer/retailer); and (ii) the selling price, the advertising level and the supply chain’s total (variable) profit per cycle are the same regardless of the leader’s identity.

Another interesting result is that the type of advertising investment (i.e., cooperative or non-cooperative) depends on the sequence in which the different types of advertising decisions are made (i.e., the respective investment shares and the total level of investment in advertising). Moreover, at conditional equilibrium, the extent to which a given party participates in the advertising investment is independent of the model parameters; rather, it is determined by whether that party is the leader or the follower. We also find that for a given cycle length, cooperative advertising is better than non-cooperative advertising for the performance of the entire supply chain. In particular, when price and advertising investment have multiplicative effects on demand, both parties benefit from cooperative advertising, and thus it is adopted naturally. Under additive effects of price and advertising, however, only the leader benefits from cooperative advertising. Notably, the dominance of cooperative advertising over non-cooperative advertising might be eliminated when the cycle length is a decision variable.

2. Literature review

Our model adds to a recent stream of literature considering age-dependent demand in combination with EOQ models. This stream includes studies by Avinadav and Arponen (2009), who considered an EOQ model in which the demand rate is polynomial in the product’s remaining time-to-expiry, and by Avinadav et al. (2013, 2014b), who extended that model to include the effect of price on the demand rate. Valliathal and Uthayakumar (2011) and Mailhami and Kamalabadi (2012) investigated a model where the demand rate is linear in price and exponential in the product’s age with partial backlogging. Avinadav et al. (2017a) considered dynamic decisions of a retailer who seeks to determine the selling price and promotion expenditures associated with a perishable product, as well as to set the order quantity and the inter-replenishment time. Dobson et al. (2017) took into account consumers’ assessment of a product’s quality over the course of its lifetime, and assumed that the demand rate is a linearly decreasing function of the age of the product. Feng et al. (2017) proposed an inventory model that stipulates the demand explicitly in a multivariate function of price, freshness and displayed stocks. Demirag et al. (2017) studied inventory ordering policies for products that attract demand at a decreasing rate as they approach the end of their useable lifetime. San-José et al. (2018) analyzed an inventory model for items whose demand rate multiplicatively combines the effects of a time-power function and a price-logit function. Whereas all these studies addressed the case of a single-echelon supply chain, the innovation of our study is in taking into account the interaction between two parties in a supply chain using a game theoretic approach.

Our use of a game theoretic framework enables us to provide insights regarding decision rights allocation in supply chains for perishable products. Broadly, decision rights allocation in a supply chain is defined as responsibility assignment among the various stakeholders regarding operational, marketing and financial decisions that have the potential to improve channel efficiency (see Tsay, 1999). Prior research on decision rights allocation includes the work of Mishra and Raghunathan (2004), who studied the allocation of responsibility for unsold inventory, including returns and price protection policies. Savaskan and Van Wassenhove (2006) compared the efficiency of a direct product collection system (in which the manufacturer collects used products) and an indirect system (in which the retailers act as product return points). Huang and Qu (2008) considered a three-echelon supply chain in which each member can configure its own supply chain by choosing members from its respective upstream supply chain. Jacobs and Subramanian (2012) examined decision rights allocation in the context of product collection and recycling, considering two scenarios: (i) responsibility for product recovery rests solely with the downstream echelon; and (ii) responsibility is shared between the downstream and the upstream echelons. Feng and Zhang (2014) analyzed a two-stage modular assembly system with two competitive suppliers, where the two suppliers either independently choose their inventory policies or are integrated into a module supplier. Jin et al. (2015) investigated the implications of assigning the responsibility and cost for sales promotion to the manufacturer or to the retailer under a wholesale price contract and under a consignment contract with revenue sharing. Chen et al. (2017) considered decision rights regarding a free after-sales service in a two-echelon supply chain facing random demand. Chernonog and Avinadav (2019) considered three wholesale price contracts for a single-period interaction between a manufacturer and a retailer, in which the responsibility for investing in advertising of a perishable product is borne either exclusively by one of the parties or by both. In this study, we investigate decision rights allocation with respect to advertising and EOQ replenishment policy for a perishable product. Specifically, we consider and compare multiple scenarios in which one party—either the manufacturer or the retailer, who might be either the leader of the supply chain or the follower—determines the amount of money to be invested in advertising, whereas the other determines the share of advertising to be borne by each party. We further assume that the leader of the supply chain sets the cycle length.

This analysis enables us to address the question of channel power, that is, who should lead the supply chain. The topic of channel power has attracted a great deal of attention in the Operations Management literature in recent years (see, e.g., Chung and Lee, 2017). It is known that choosing a certain player as the leader can lead to supply chain coordination (see Avinadav et al., 2014a). In this study, we focus on the case of asymmetric power balance between a manufacturer and a retailer, and consider two models. The first model assumes the manufacturer is the leader, reflecting, for example, the case of Unilever (Wadlow, 2017), a supplier of consumer goods (food, beverages, etc.) that dominates the market and is much larger than most of the retailers selling its products. The second model assumes the retailer is the leader; this scenario corresponds, for example, to the case of Walmart (https://www.walmart.com/), a leading retailer that is much larger and more dominant than most of its suppliers. Recent studies have investigated the effect of supply chain leadership in different contexts. For example, Edirisinghe et al. (2011) investigated the implications of channel power for supply chain stability in a setting where multiple suppliers sell substitutable products through a common retailer. SeyedEsfahani et al. (2011) established four game-theoretic models to study the effect of supply chain power balance on the optimal decisions of the supply chain members. Choi et al. (2013) investigated a closed-loop supply
chain, which consists of a retailer, a collector, and a manufacturer, and examined its performance under different power structures. Chiu et al. (2016) studied the effects of channel leadership and information asymmetry on the coordination of a two-echelon supply chain, which sells a product with a general stochastic price-dependent-demand function. A recent study by Dennis et al. (2017) investigated the impact of market power and retail channel dominance on a manufacturer’s optimal distribution channel strategy.

Our work also provides new insights regarding the tradeoffs between cooperative versus non-cooperative advertising. Non-cooperative advertising refers to the common practice in which one party takes nearly exclusive responsibility for the advertising. For example, some dominant retailers, such as Walmart and Target, frequently advertise certain products, and the suppliers of these products do not have to advertise at all (Liu et al., 2014). In contrast, Mengniu, an Asian manufacturer of dairy products, handles all advertising activities, and prohibits its retailers from doing so (Ni, 2007). Cooperative advertising, in turn, is a cost allocation mechanism (Shreve, 2018) in which multiple supply chain members share responsibility for advertising. This approach is accumulating popularity: According to He et al. (2009), the total expenditure on cooperative advertising in 2000 was estimated at $15 billion, compared with $900 million in 1970.

Cooperative and non-cooperative advertising have been widely discussed in the Operations Management literature. For example, SeyediEsfahani et al. (2011) considered vertical cooperative local advertising along with national advertising and pricing decisions in a two-echelon supply chain. Aust and Buscher (2014) reviewed cooperative advertising models in supply chain management, identifying five different definitions of cooperative advertising that are used in the operations research literature. Two papers by El Ouardighi et al. (2016a, 2016b) introduced a model of a monopolist firm in which sales are investigated a multi-item EOQ model in a two-echelon supply chain, which consists of a retailer, a collector, and a manufacturer, and interact using a wholesale price contract. Specifically, under this contract, the manufacturer sets a wholesale price \( w \), and the retailer sets her unit margin \( m \), such that the selling price is dictated by both parties and equals \( p = w + m \). Demand is accelerated by advertising, which is funded according to the following arrangement: one party sets the participation shares: \( \theta = (0,1) \) for the manufacturer and \( 1 - \theta \) for the retailer, whereas the other party sets the overall level of advertising, \( s \). Note that \( \theta = 0 \) or \( \theta = 1 \) implies “non-cooperative” advertising, i.e., only one party bears the advertising investment (see, for example, Hu et al., 2019), whereas \( \theta e(0,1) \) implies “cooperative” advertising, i.e., both parties invest in advertising (see, for example, Chernonog and Avinadav, 2019; Chen, 2018). The demand rate, denoted by \( \lambda(p, s, t) \), is affected by three factors: the selling price \( p \), the advertising level \( s \), and the age of the product \( t \). Whereas the first two factors are directly controlled by the parties, the age of the product is only partially controlled, via the replenishment policy. The parties apply an EOQ policy: an order of fixed size is made every fixed time interval \( T \) (‘cycle length’).

We assume that no backlogging occurs. This assumption is based on the notion that, when purchasing a given product type, the typical customer is likely to prefer a certain brand (in our case, the preferred product is the focal product); however, if that product is out of stock, the customer will by a substitutional available product (Trautrimas et al., 2009; Minner and Transchel, 2017). This idea is particularly likely to be applicable to common perishable products, which are usually purchased to satisfy immediate needs (see, e.g., Krishna, 1992; You, 2005, Tsao and Sheen, 2008, Avinadav and Arponen, 2009, Avinadav, 2018): For example, a customer who arrives at a supermarket for a weekly shopping trip and cannot find his/her preferred brand of milk, bread or ice-cream will probably compromise and buy another brand instead of returning at replenishment time especially for that product.

We adopt a deterministic EOQ model as the framework for our research. Although in reality demand can almost never be forecasted with certainty, deterministic models are generally accepted as being of value towards investigating certain features of real-world problems. According to Hadley and Whitin (1963, p. 29), “the results obtained from these models yield, qualitatively, the proper sort of behavior—-even when the deterministic demand assumption is removed”. As is shown in Proposition 1 of Avinadav et al. (2014b), herein, the optimal time for replenishment is exactly when inventory is exhausted. Consequently, under the optimal ordering policy, the retailer’s order quantity for given \( T, p \) and \( s \) is equal to the total quantity demanded during the entire cycle, \( \int_0^T \lambda(p, s, t) \, dt \). In what follows we assume that the optimal ordering policy is implemented.

The investment rate in advertising is \( C(s) \), which is an increasing function of \( s \). The retailer bears the holding cost of inventoried units (\( h \) per unit per unit of time), whereas the manufacturer bears the production cost (\( c \) per unit). Consequently, the retailer’s and manufacturer’s variable profits per cycle are, respectively,

\[
\Pi_b(T, \theta, w, m, s) = -(1-\theta)T \cdot C(s) + \int_0^T (m - h t) \lambda(p, s, t) \, dt, \tag{1}
\]

\[
\Pi_d(T, \theta, w, m, s) = -\theta T \cdot C(s) + (w - c) \int_0^T \lambda(p, s, t) \, dt. \tag{2}
\]

In addition, we assume that the leader of the supply chain bears a fixed order cost per cycle, \( K_i, i \in [R, M] \) (for example, part of a transportation cost, cost of arranging units on shelf, time and effort costs to make an order). We further assume that, because the leader incurs this cost, he or she has the right to set the cycle length.

We investigate two sets of scenarios that differ in the identity of the party who is the leader. To provide a unified model formulation for the two scenarios, we introduce a binary variable \( \beta \), which equals 1 if the retailer is the leader, and 0 if the manufacturer is the leader. We further
Table 1: Sequence of decisions according to the various models.

<table>
<thead>
<tr>
<th>Leadership Investment splitter</th>
<th>Manufacturer leader ((\beta = 0))</th>
<th>Retailer leader ((\beta = 1))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Manufacturer sets the investment shares</strong></td>
<td><strong>Model 1</strong></td>
<td><strong>Model 3</strong></td>
</tr>
<tr>
<td>1. Manufacturer sets (T, w, w) and (\theta)</td>
<td>1. Retailer sets (T, m) and (s)</td>
<td>1. Retailer sets (T, m) and (\theta)</td>
</tr>
<tr>
<td>2. Retailer sets (m) and (s)</td>
<td>2. Manufacturer sets (w, w) and (\theta)</td>
<td>2. Manufacturer sets (w, w) and (\theta)</td>
</tr>
<tr>
<td><strong>Retailer sets the investment shares</strong></td>
<td><strong>Model 2</strong></td>
<td><strong>Model 4</strong></td>
</tr>
<tr>
<td>1. Manufacturer sets (T, w, w) and (s)</td>
<td>1. Retailer sets (T, m) and (\theta)</td>
<td>1. Retailer sets (T, m) and (\theta)</td>
</tr>
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</tr>
</tbody>
</table>

Note that a leader can choose who will set the investment share and who will set the overall level of advertising by choosing between Model 1 and Model 2 when the manufacturer is the leader, or by choosing between Model 3 and Model 4 when the retailer is the leader. Thus leadership affects decision rights allocation.

define the long-run average profit rates of the parties as follows:

\[
\eta_i(T, \theta, w, m, s) = (\Pi_i(T, \theta, w, m, s) - BK_i)/T
\]

and

\[
\eta_i(T, \theta, w, m, s) = (\Pi_i(T, \theta, w, m, s) - (1 - B)K_M)/T.
\]

Note that the leader wishes to maximize the average profit rate, whereas the follower wishes to maximize the profit per cycle, because the cycle length has already been determined by the leader.

For each leadership scenario (manufacturer-leader and retailer-leader), we investigate two models that differ in the identity of the party who sets the investment shares; thus, we investigate four different models. Table 1 presents the sequence of the decisions in each of the four models. Note that each model is studied under two different forms of the marketing effect on demand (see section 3.3), so, in total, we investigate eight cases. We formulate the interaction between the manufacturer and the retailer as a Stackelberg game, which is solved using backward induction.

Following the sequence of decisions in all four models (in which the cycle length is set by the leader), the problems can be solved in two optimization stages. In the first stage, we assume that \(T\) is given and find conditional equilibrium as a function of the cycle length; this equilibrium is denoted by \((w^T_i, m^T_i, s^T_i, \theta^T_i, T)\). Let \(\Pi^T_i(T) = \Pi_i(w^T_i, m^T_i, s^T_i, \theta^T_i, T)\) be the variable profit per cycle at conditional equilibrium of party \(i, i \in [R, M]\). In the second stage, we find the value of \(T\) that maximizes the leader’s long-run average profit by solving

\[
\max_T \left\{ \eta_i(T) = \frac{\Pi^T_i(T) - K_i}{T} \right\}, i \in [R, M]
\]

where \(i\) is the index of the leader. We use this method of solution because, as we show in the next sections, the conditional equilibrium has a closed-form solution, whereas finding the optimal value of \(T\) requires numeric methods. Another benefit of maximizing the objective using these two stages is that in the real world, the cycle length for replenishment may be set exogenously, which means that it cannot be controlled for a specific product. This may be the case, for example, when a retailer uses a joint replenishment policy, in which she receives the product in a shipment together with several other products (Chernonog and Goldberg, 2018), even if each product has a different optimal cycle length. According to Sazvar et al. (2016), such a policy allows for economies of scale and savings in various inventory costs such as transportation and other replenishment-related costs. Another example is a manufacturer who uses a single truck delivery to ship a product to several retailers in different locations, so the cycle length is set by joint optimization (see e.g., Banerjee et al., 2003; Kogan et al., 2008; Darvish and Odah, 2010). In both examples the marketing decisions are being set by local optimization for each product or for each retailer separately. Therefore, investigating the conditional equilibrium for a given cycle length has its own practical merit.

Note that the second-stage optimization problem in (3) is identical across all four models. The first-stage optimization problem is different for each model, as is detailed below.

**Model 1:**

\[
\max_{\theta, w} \eta_i(T, \theta, w, m(T, \theta, w), s(T, \theta, w))
\]

\[
\text{s.t. } (m(T, \theta, w), s(T, \theta, w)) = \arg\max_{m, s}\eta_i(T, \theta, w, m, s)
\]

**Model 2:**

\[
\max_{w, s}\eta_i(T, \theta(T, w, s), w(T, w, s), m(T, w, s), m, s)
\]

\[
\text{s.t. } (\theta(T, w, s), m(T, w, s)) = \arg\max_{\theta, m}\eta_i(T, \theta, w, m, s)
\]

**Model 3:**

\[
\max_{m, s}\eta_i(T, \theta, w, m, s(T, \theta, m), m, s)
\]

\[
\text{s.t. } (w(T, \theta, m), s(T, \theta, m)) = \arg\max_{w, s}\eta_i(T, \theta, w, m, s, \theta, T)
\]

**Proposition 1.** The manufacturer’s equilibrium participation share in advertising under Model 2 is \(\theta^F = 1\); and under Model 3 it is \(\theta^F = 0\).

**Proof.** By (1), \(\Pi_i(w, m, s, \theta, T)\) is an increasing function of \(\theta\), so \(\theta(T, w, s) = 1\) in Model 2; by (2), \(\Pi_i(w, m, s, \theta, T)\) is a decreasing function of \(\theta\), so \(\theta(T, m, s) = 0\) in Model 3. Thus, the claim is proved.

It is clear that once the advertising level (s) has been set by the party that moves first, the follower has no motivation to participate, because any participation will not increase its income but will increase its costs. Consequently, the sequence of decisions above leads to non-cooperative investment in advertising. In sections 4.1 and 4.2 it is proved that under Models 2 and 3 equilibrium is obtained by choosing \(\theta \in (0,1)\). Thus, hereafter, we refer to Models 1 and 4 as cooperative investment models, and to Models 2 and 3 as non-cooperative investment models.

**3.2. The demand function**

In line with Valliathal and Uthayakumar (2011), Maihami and Kamalabadi (2012), Avinadav et al. (2013, 2014b), Feng et al. (2017), and Chernonog and Avinadav (2019), we assume that the effects of marketing and of product age on demand (denoted \(f(p, s)\) and \(g(t)\), respectively) follow the multiplicative form:

\[
\lambda(p, s, t) = f(p, s)g(t).
\]

Our choice of the multiplicative demand form is motivated by the results of Avinadav et al. (2014b), according to which a genuine additive model incorporating an age effect does not exist, since when the product expires demand vanishes regardless of its selling price or advertising.

As discussed in the introduction, customers purchasing perishable
products prefer to buy fresh items, so when products age demand decreases. We capture this tendency by assuming that \( g(t) \) is a non-increasing function, i.e., \( \frac{d}{dt} g(t) \leq 0 \leq t \leq t_1 \), and that \( \lim_{t \to \infty} g(t) = 0 \) (i.e., in the long run, demand vanishes due to deterioration). In addition, without loss of generality, assume \( 0 \leq g(t) \leq 1 \) for any value of \( t \) and \( g(0) = 1 \), so \( f(p,s) \) is the demand rate for fresh products (with age zero).

For simplicity of presentation, let \( \ell_T = \int T g(t) dt \) be the age effect on the demand over the entire cycle, which is an increasing function of \( T \). By the definition of \( \ell_T \), \( \phi_T = \ell_T / T \) is the average age effect on the demand rate over the entire cycle. In addition, let \( \eta_T = \int T g(t) dt \) be the average time a unit spends on the shelf before being sold. In order to avoid losses of each party, we require

\[
w \geq c \text{ and } m \geq h_T 
\]

Hereafter, we define \( w - c \) and \( m - h_T \) as the net unit profit margins of the manufacturer and the retailer, respectively.

The following lemma presents some general properties regarding the age effect on the demand.

**Lemma 1.** (i) \( \phi_T \) is a decreasing function of \( T \); (ii) \( \lim_{T \to \infty} \phi_T = 0 \); (iii) \( \lim_{T \to 0} \phi_T = 0 \) and \( \phi_T \) is an increasing function of \( T \).

**Proof:** See Appendix.

Now we investigate the four models discussed above under the multiplicative demand form between the marketing effect and the age effect as given in (10). Let \( \mu^T(T) = \mu^T_0(T) + \mu^T_{01}(T) \) be the total supply chain profit and \( p_E^T \equiv w_E^T + m_E^T \) be the selling price of the product at conditional equilibrium.

**Proposition 2.** For a given type of advertising (cooperative/non-cooperative), at conditional equilibrium:

(i) The net unit profit margin of each party is determined only by the party's role (leader/follower) and not by its identity (manufacturer/retailer);

(ii) The selling price and advertising level are the same regardless of the leader's identity.

**Proof:** See Appendix.

Put differently, **Proposition 2(i)** implies that \( m^T_E - h_T \) in Model 1 (Model 2) is equal to \( w_E^T - c \) in Model 4 (Model 3). Similarly, \( w_E^T - c \) in Model 1 (Model 2) is equal to \( m_E^T - h_T \) in Model 4 (Model 3). By **Proposition 2(ii)**, \( p_E^T \equiv w_E^T + m_E^T \) is the same regardless of the leader's identity.

**Theorem 1.** For a given type of advertising (cooperative/non-cooperative), at conditional equilibrium:

(i) The variable profit per cycle of each party is determined only by the party's role (leader/follower) and not by its identity (manufacturer/retailer);

(ii) The supply chain's total variable profit per cycle is the same regardless of the leader's identity.

**Proof:** See Appendix.

Thus, **Theorem 1(i)** implies that \( I^T_0(T) \) in Model 1 (Model 2) is equal to \( I^T_0(T) \) in Model 4 (Model 3), and that \( I^T_{01}(T) \) in Model 1 (Model 2) is equal to \( I^T_{01}(T) \) in Model 4 (Model 3). **Theorem 1(ii)**, in turn, implies that \( \mu^T(T) \) in Model 1 (Model 2) is equal to that in Model 4 (Model 3). Thus, when the cycle length is given, Pareto improvement cannot be achieved by switching roles in the supply chain.

The results of **Proposition 2** and **Theorem 1** follow from the combination of the multiplicative form of the demand and the similar cost structures of the parties. Note that this result is valid for any form of the marketing effect on the demand \( f(p,s) \) and for any investment rate \( C(s) \).

**Corollary 1.** For a given type of advertising (cooperative/non-cooperative), if \( K_M = K_R \) the equilibrium cycle length is independent of the identity of the leader.

**Proof:** Straightforward from **Theorem 1(i)** and (5).

**Proposition 3.** If \( \pi(T) \in [R, M] \) is a quasi-concave function then a larger fixed order cost leads to a longer equilibrium cycle length.

**Proof:** See Appendix.

Hereafter, we assume that the retailer's and the manufacturer's profit rate functions are quasi-concave. The following proposition orders the equilibrium cycle lengths in Models 1 and 4 (Models 2 and 3) according to the leader's fixed order cost.

**Proposition 4.** For a given type of advertising (cooperative/non-cooperative), a leader who has a larger fixed order cost sets a longer cycle length.

**Proof:** See Appendix.

By **Proposition 4**, although the follower does not dictate the cycle length, he or she can affect its value by choosing a leader with a certain fixed order cost. From the customers' point of view, giving the decision right about the cycle length to the party with the lower replenishment cost is preferable in terms of ensuring greater product freshness.

### 3.3. The marketing effect on demand

The marketing effect on the demand can take different forms, which can lead to significantly different results (see, e.g., Gal-Or, 1985; Lau and Lau, 2003; Wang et al., 2013, Avinadav et al., 2014a,). Herein, we consider both of the two most popularly-assumed demand forms: additive, denoted by \( f_1(p,s) \), and multiplicative, denoted by \( f_2(p,s) \). According to the first form, which is implemented by Ma et al. (2013), Avinadav et al. (2017b), Avinadav et al. (2018), and Chernonog (2018),

\[
f_1(p,s) = a_1 - \alpha p + s, 
\]

where \( a_1 \) is a base demand, and \( a_1 \) is the customer's price sensitivity for a fresh product \( t = 0 \). This form is useful when each factor has an absolute effect on the demand, so that the reservation price can be controlled by the advertising level. This form of demand suits products with low levels of substitutability (e.g., the willingness to pay for organic food is higher than that of non-organic food).

According to the second form of demand, which is implemented by Aust and Buscher (2012), Chernonog et al. (2015), Avinadav et al. (2017a, and Chernonog et al. (2019),

\[
f_2(p,s) = (a_2 - a_2 p)s, 
\]

where \( a_2 \) is a scale factor and \( a_2 / a_2 \) is the reservation price. This form is useful when each factor has a relative effect on the demand for any given level of the other effect. Since this demand form is characterized by a fixed reservation price, which is independent of the advertising level, it suits products with high levels of substitutability (e.g., different brands of mineral water, toilet paper, milk etc.). The main difference between the two forms of the marketing effect on the demand is that in the additive form the effect of each factor is independent of the other factor, whereas in the multiplicative form, the effects are mutually dependent and include interactions.

### 3.4. The cost of advertising

For the remainder of our analysis, we assume that the investment rate in advertising takes the following form:

\[
C(s) = \gamma s^2/2, 
\]

where \( \gamma \) is a scale parameter corresponding to investment efficiency.
such that a lower value of \( \gamma \) implies a more effective investment. The assumption of a quadratic cost function reflects the law of diminishing returns, and is commonly used in the operations management literature (see, e.g., Lau et al., 2010; Ma et al., 2013; Wang et al., 2013). Note that both \( s \) and \( \gamma \) have different unit measures under the additive and multiplicative demand forms; for example, under the additive demand form, \( s \) is measured in demand units, whereas under the multiplicative form \( s \) is dimensionless.

4. Conditional equilibrium analysis

4.1. Additive form of the marketing effect on demand

In this section, we find and investigate the conditional equilibrium of Models 1–4 under the assumption that the marketing effect on demand takes an additive form as in (12). We denote the equilibrium for each model by using the superscripts \( A_1, A_2, A_3 \) or \( A_4 \), respectively. For simplicity of presentation, let \( \lambda_1 \equiv a_1 - a_2(c + h_T) \) be the demand at cost price (purchase plus holding) and no advertising at all. We give the explicit expressions of the conditional equilibrium and other measures in Table 2; the proofs of these expressions are given in the Appendix. Comparing the results of Models 1 and 4 (Models 2 and 3) supports the claims in Proposition 2 and Theorem 1 that the measures are independent of the leader's identity for a given type of advertising (cooperative/non-cooperative).

As shown in the first row of Table 2, for each model, there is a necessary and sufficient condition for conditional equilibrium to exist. Note that, if this condition does not hold, then at least one party has an infinitely large profit (see proof in the Appendix). Specifically, when the scale parameter of the investment efficiency \( \gamma \) is small enough, advertising is insufficiently costly, so it is economically viable to use an infinitely large price and advertising levels. Similar arguments are given in Savaskan et al. (2004), Avinadav et al. (2017b) and Chernonog and Avinadav (2019). By Table 2, the conditional equilibrium exists iff \( \gamma > \frac{3}{4a_1} \) under Models 1 and 4, and iff \( \gamma > \frac{1}{4a_1} \) under Models 2 and 3. This condition depends on the cycle length \( T \), which is a decision variable whose value is determined in the second step of optimization. In order to ensure that equilibrium exists for any cycle length under the additive demand form, following (i) and (ii) of Lemma 1, we require for Models 1 and 4:

\[
\gamma > \frac{3}{4a_1}, \quad (15)
\]

and for Models 2 and 3:

\[
\gamma > \frac{1}{4a_1}. \quad (16)
\]

Comparing (15) and (16), we observe that the condition under which conditional equilibrium exists is stronger under cooperative investment in advertising (Models 1 and 4) than under non-cooperative investment (Models 2 and 3). This result can be explained as follows: compared with cooperative investors, a sole investor has to be more efficient (lower \( \gamma \)) to obtain infinitely large profits.

The following observations emerge from Table 2.

**Observation A1.** (i) The advertising level is a decreasing function of the cycle length. (ii) The advertising level under cooperative investment is higher than that under non-cooperative investment by a ratio of at least 1.5, where the ratio decreases in the cycle length.

**Proof:** See Appendix.

**Observation A2.** The customers enjoy a lower selling price under non-cooperative advertising than under cooperative advertising.

**Proof:** See Appendix.

**Observation A3.** The demand under cooperative investment is higher than that under non-cooperative investment up to a ratio of \( 2^2 \), where the ratio decreases in the cycle length.

**Proof:** See Appendix.

**Observation A4.** The parties’ investment shares are independent of the model parameters regardless of the investment type (cooperative/non-cooperative), implying that the conditional equilibrium investment shares are equal to the unconditional ones.

**Observation A5.** (i) The leader’s profit under cooperative investment is higher than that under non-cooperative investment up to a ratio of \( 2^2 \), and the ratio decreases in the cycle length. (ii) The follower’s profit under cooperative advertising is lower than that under non-cooperative advertising, where the ratio between these two profit values increases towards 1 when the cycle length increases.

**Proof:** See Appendix.

<table>
<thead>
<tr>
<th>Model A1</th>
<th>Model A2</th>
<th>Model A3</th>
<th>Model A4</th>
</tr>
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<tbody>
<tr>
<td>Manufacturer leader</td>
<td>Manufacturer leader</td>
<td>Retailer leader</td>
<td>Retailer leader</td>
</tr>
<tr>
<td><strong>Cooperative investment</strong></td>
<td>Non-cooperative investment</td>
<td>Non-cooperative investment</td>
<td>Co-operative investment</td>
</tr>
<tr>
<td>Condition for equilibrium existence</td>
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<tr>
<td>( y &gt; \frac{3}{4a_1} )</td>
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<tr>
<td>( \frac{a_2}{a_1} \geq \frac{y}{\gamma} )</td>
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</tr>
<tr>
<td>( h_T + \frac{a_2}{a_1} \geq \frac{y}{\gamma} )</td>
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<tr>
<td>( c + \frac{a_2}{a_1} \geq \frac{y}{\gamma} )</td>
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<tr>
<td>( \frac{1}{3} )</td>
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<td>( \frac{1}{16} )</td>
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<tr>
<td>( \frac{h_T + a_2}{a_1} )</td>
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<tr>
<td>( c + \frac{a_2}{a_1} )</td>
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</tbody>
</table>
Observation A6. The total profit of the supply chain under cooperative investment is higher than that under non-cooperative investment, and the ratio between these two profit values decreases towards 1 when the cycle length increases.

Proof: See Appendix.

Observation A7. A cooperative advertising will be chosen since the leader dictates the investment policy: cooperative or non-cooperative.

Proof: See Appendix.

4.2. Multiplicative form of the marketing effect on demand

In this section, we find and investigate the conditional equilibrium of Models 1–4 under the assumption that the marketing effect on demand takes a multiplicative form as in (13). We denote the equilibrium for each model by the superscripts M1, M2, M3 or M4, respectively. For simplicity of presentation, let $\theta_x \equiv \alpha_x - \alpha_x(c + h_x)$ be the demand of advertising (purchase plus holding) and no advertising at all. We give the explicit expressions of the conditional equilibrium and other measures in Table 3; the proofs are given in the Appendix. Comparing the results under Models 1 and 4 (Models 2 and 3) supports the claims given in Proposition 2 and Theorem 1 that the measures are independent of the leader’s identity for a given type of advertising (cooperative/non-cooperative).

Observation M1. (i) The advertising level is a decreasing function of the cycle length. (ii) The advertising level under cooperative investment is higher than that under non-cooperative investment by a factor of $\frac{3}{\alpha}$.

Observation M2. The customers enjoy a lower selling price under cooperative investment compared with the selling price under non-cooperative investment.

Observation M3. The demand under cooperative investment is higher by a factor of $\frac{3}{\alpha}$ than that under non-cooperative investment.

Observation M4. The parties' investment shares are independent of the model parameters regardless of the investment type (cooperative/non-cooperative), implying that the conditional equilibrium investment shares are equal to the unconditional ones.

Observation M5. (i) The leader's profit under cooperative investment is higher by a factor of $\frac{3}{\alpha}$ than that under non-cooperative investment.

(ii) The follower's profit under cooperative investment is higher by a factor of $\frac{3}{\alpha}$ than that under non-cooperative investment.

Observation M6. The total profit of the supply chain under cooperative investment is higher by a factor of $\frac{3}{\alpha}$ than that under non-cooperative investment.

Observation M7. A cooperative investment dominates non-cooperative investment from the perspectives of the parties and customers.

Proof: See Appendix.

4.3. Comparison of the results under the two forms of the marketing effect on demand

An interesting observation emerges from a comparison between the results shown in Tables 2 and 3: Under the additive form of the marketing effect on demand, conditional equilibrium exists only when advertising is sufficiently costly, whereas under the multiplicative demand form, conditional equilibrium always exists. This result can be explained as follows: under the additive demand form, both $s$ and $p$ can approach infinity, resulting in infinitely large demand, whereas under the multiplicative demand form the unit selling price $p$ is limited by a finite reservation price, so for large values of $s$ the cost of advertising is higher than its contribution to the revenue. Another interesting result is that both the retailer and the manufacturer adopt a cost-plus pricing strategy, regardless of the advertising type (cooperative/non-cooperative) and demand form.

Clearly, the advertising level is higher under cooperative investment than under non-cooperative investment for both forms of the marketing effect on demand (see observations A1 and M1). However, it can be seen that under the additive form of the marketing effect on demand, the customers enjoy a lower selling price under non-cooperative investment than they do under cooperative investment, whereas under the multiplicative demand form the opposite occurs (see observations A2 and M2). Note that the demand is higher under cooperative investment than under non-cooperative investment for both demand forms (see observations A3 and M3). Thus, at equilibrium, the influence of advertising on demand is more significant than the influence of the selling price.

Under both demand forms it is better for the leader to use cooperative investment rather than non-cooperative investment, whereas the follower’s preference depends on the demand form (see...
observations A5 and M5). Following observations A7 and M7, the cooperative investment in advertising will be chosen.

5. Equilibrium cycle length

In order to satisfy the requirements in (11) combined with the conditional equilibrium in Tables 2 and 3, and the definitions of $T_1$ and $T_2$, $T$ should be restricted to satisfy

$$ T \leq \frac{(a_i/c_i) - c}{h}, $$

where $j = 1$ is used for the additive form of the marketing effect on demand, and $j = 2$ is used for the multiplicative form. Since $T_j$ is an increasing function of $T$ (see Lemma 1(iii)), constraint (17) can be written as

$$ T \leq T_j, $$

where $T_j$ either satisfies $T_j = \frac{(a_i/c_i) - c}{h}$ or is infinitely large. Note that $T_0 = 0$ or $\theta_T = 0$ implies $T_j = 0$, $m_j^T = h_T$ and $w_j^T = c$, which, in turn, imply that the leader has zero variable profits per cycle and thus suffers loss equals to the fixed replenishment cost; and the follower has zero profit. Thus, it is not profitable for the two parties to interact when $T \geq T_j$, due to high average storage cost $h_T$.

Proposition 5. The equilibrium $T^E = \arg\max_{T \in \mathbb{T}} \{\mathbb{E}_i^T(T - K_i)/T\}$, where $i \in \{L, F\}$ is the index of the leader, $j = 1$ is used for the additive form of the marketing effect on demand, and $j = 2$ is used for the multiplicative form.

Proof. Straightforward from (5) and (18).

The equilibrium cycle length affects all decisions of both parties except for the parties’ investment shares (see Tables 2 and 3). Due to the complexity of obtaining a closed-form expression for the equilibrium value of the cycle length ($T^E$), it is necessary to obtain this value numerically. Accordingly, in order to analyze the effect of the model parameters on the equilibrium (non-conditional), we use numerical examples, as detailed in the next section.

6. Numerical examples

We present several numerical examples to demonstrate how the parties’ strategies and their profits at equilibrium are affected by the fixed ordering cost and the age effect on demand. For simplicity of analysis, we use $g(t) = \begin{cases} 1 - (t/E)^n & t \leq E \\ 0 & t > E \end{cases}$, where $E$ is the product’s expiration date, and $n$ is the shape parameter of the demand decrease in time ($n > 0$). Note that a higher value of $n$ implies a weaker effect of the product’s age on demand (for further discussion of this function see Avinadav et al., 2013). Two numerical examples are analyzed: one for the additive form of the marketing effect on demand (Models A1-A4), and the other for the multiplicative form (Models M1-M4). Following Proposition 2 and Theorem 1: (i) We refer to Models A1 and A4 (M1 and M4) as cooperative advertising models, whereas we refer to Models A2 and A3 (M2 and M3) as non-cooperative advertising models; and (ii) we focus on the distinction between the leader and the follower instead of the distinction between the manufacturer and the retailer. By presenting the analysis in this way, we are able to capture the results for all eight models with two examples.

The example for the additive form of the marketing effect on demand is based on the following parameter values: $a_1 = 1000$; $a_2 = 2$; $c = 60$; $h = 0.1$; $\gamma = 4.375$; $E = 10$ and $n = (0.5, 1.5)$ under an additive form of the marketing effect on demand with cooperative and non-cooperative advertising.

Fig. 1. Supply chain measures for $a_1 = 1000$; $a_2 = 2$; $c = 60$; $h = 0.1$; $\gamma = 4.375$; $E = 10$ and $n = (0.5, 1.5)$ under an additive form of the marketing effect on demand with cooperative and non-cooperative advertising.
The example for the multiplicative form of the marketing effect on demand is based on the following parameter values:

\[ c = 60; \ h = 0.1; \gamma = 4.375; \ E = 10 \text{ and } n = \{0.5, 1.5\}, \] and is presented in Fig. 1.

Fig. 1(a) supports Proposition 3, according to which the cycle length increases in the fixed ordering cost. The cycle length under cooperative advertising is shorter than that under non-cooperative advertising. As expected, when customers' sensitivity to freshness is lower (i.e., the value of \( n \) is higher), the cycle length is longer. By Fig. 1(b), the advertising level decreases in the fixed ordering cost and in customers' sensitivity to freshness (i.e., it increases in \( n \)). Naturally, the advertising level is higher under cooperative advertising. By Fig. 1(c), (e) and (g), a higher fixed ordering cost results in a lower wholesale price and a lower retailer's margin (and thus also a lower selling price), which means that the parties bear the increase in the fixed ordering cost without sharing it with the customers. Consequently, the profit rate of each party (and thus also the total supply chain) decreases in the fixed ordering cost (see Fig. 1(d), (f) and (h)). We further see that profits of the leader and of the entire supply chain are greater under cooperative advertising than under non-cooperative advertising, whereas the follower is better off with non-cooperative advertising. Note that the observations under conditional equilibrium, as given in Section 3, are found to be valid in our example also for unconditional equilibrium.

The example for the multiplicative form of the marketing effect on demand is based on the following parameter values: \( a_2 = 100; \alpha_2 = 0.8; c = 60; \ h = 2.2; \gamma = 200; \ E = 20 \text{ and } n = \{0.5, 1.5\}, \) and is presented in Fig. 2.

As in our analysis of the additive form of the marketing effect on demand, we observe that Fig. 2(a) supports Proposition 3, according to which the cycle length increases in the fixed ordering cost, and the cycle length under cooperative advertising is shorter than that under non-cooperative advertising. However, in contrast to the additive case, a higher level of customer sensitivity to freshness (a lower value of \( n \)) does not necessarily translate into a longer or shorter cycle length. By Fig. 2(b), and similarly to the case of the additive form of the marketing effect on demand, the advertising level decreases in the fixed ordering cost and in customers' sensitivity to freshness (increases in \( n \)). Naturally, the advertising level is higher under cooperative advertising. By Fig. 2(e) and (g), a higher fixed ordering cost results in a lower unit profit margin of the leader and a higher unit profit margin of the follower. By Fig. 2(c), the selling price increases when the fixed ordering cost increases, which means that the leader and the customers bear the increase in the fixed ordering cost, whereas the follower uses it to increase its unit profit margin. However, the increase in the follower's unit profit margin does not ensure an increase in its profit rate, which actually decreases in \( K \) (see Fig. 2(h)).

Similarly, the profit rates of the leader and of the entire supply chain decrease in the fixed ordering cost (see Fig. 2(f) and (d)). By comparing the profit rates under cooperative versus non-cooperative advertising, we observe that for the leader there is no dominant advertising type, whereas for the follower cooperative advertising is better, as shown in Fig. 2(f) and (h). This result is in contrast to the dominance of
cooperative advertising over non-cooperative advertising for a given cycle length (see Observation M5). In addition, cooperative advertising is better than non-cooperative one for maximizing the total profit rate of the entire supply chain (see Fig. 2(d)).

We further observe that, in contrast to the case of exogenous cycle length, under endogenous cycle length, there are cases in which Pareto improvement can be achieved by switching the roles of the leader and the follower in the supply chain. This phenomenon occurs, for example, in the following scenario: Assuming the multiplicative form of the marketing effect on demand and cooperative advertising, suppose that the manufacturer is the leader of the supply chain with fixed order cost $K = 600$, and that the shape parameter of the demand is $n = 1.5$. In such a case the manufacturer's profit rate is 417.0, whereas the retailer's profit rate is 401.0. If the manufacturer concedes his leadership and the retailer's fixed order cost is $K = 400$, the manufacturer's profit rate increases to 425.3 ($+2\%$) and the retailer's profit rate increases to 485.2 ($+21.1\%$). In contrast, when the shape parameter of the demand is $n = 0.5$ (i.e., the perishability effect is stronger), Pareto improvement cannot be achieved by switching leadership. For example, when the manufacturer is the leader of the supply chain with fixed order cost $K = 600$, the manufacturer's profit rate is 159.8, whereas the retailer's profit rate is 236.1. If the manufacturer concedes his leadership and the retailer's fixed order cost is $K = 400$, the manufacturer's profit rate increases to 275.6 ($+72.5\%$) and the retailer's profit rate decreases to 235.4 ($−0.3\%$). These observations indicate that the product's perishability affects the willingness of the parties to switch roles (leader/follower) in order to be more profitable.

7. Conclusion

In this study, we analyzed a supply chain comprising a manufacturer and a retailer of a perishable product, who are setting the terms of a wholesale price contract. We investigated eight different models, which differed according to the identity of the leader of the supply chain, the demand form, and by the type of advertising (cooperative/non-cooperative). First, we provided general results related to the parties' decisions and profits in the different models considered. In particular, we found that for a given type of advertising, at conditional equilibrium (i.e., when the cycle length is given): (i) the leader's (follower's) net unit profit margin and the variable profit per cycle are independent of the leader's identity (manufacturer/retailer); (ii) the leader's identity has no effect on the selling price, the advertising level and the supply chain's total variable profit per cycle. These two results are valid for any cost function of advertising and for any form of the marketing effect on demand. Another general result is that the choice between cooperative and non-cooperative advertising depends on which decision is made first: investment share or advertising investment level. Interestingly, although, for a given cycle length, cooperative

Fig. 2. Supply chain measures for $\alpha_2 = 100; \alpha_2 = 0.8; c = 60; h = 2.2; \gamma = 200; E = 20$ and $n = \{0.5, 1.5\}$ under cooperative and non-cooperative advertising.
advertising is always more profitable for the supply chain members, when the cycle length is a decision variable, it is possible that non-cooperative advertising will be more profitable to the leader of the supply chain. Finally, with regard to the role of perishability in equilibrium, we found that a higher level of customer sensitivity to freshness may either increase or decrease the cycle length, which is counterintuitive.

Our findings point to the following practical recommendations for supply chain members:

(i) If the cycle length is exogenously determined, then, following observations A5 and M5, the leader of the supply chain should dictate the advertising investment shares and let the follower set the advertising amount. This decision rights allocation scheme—which leads to cooperative advertising—benefits both supply chain members (as compared with schemes in which the leader dictates the advertising amount and lets the follower determine the shares).

(ii) If the cycle length is endogenously determined, then the leader should make his or her decisions on a case-by-case basis, taking into account the parameter values. In particular:

- Our results in Section 6 indicate that the leader of the supply chain should consider both options—setting advertising investment shares (leads to cooperative advertising) or setting the advertising amount (leads to non-cooperative advertising)—and select the one that is most profitable.

- Our results in Section 6 further suggest that the supply chain leader should consider conceding leadership to the follower, as doing so might yield Pareto improvement (depending on the parameter values).

- The follower, who can be either the manufacturer or the retailer, should take into account the fixed ordering cost of the leader, since this cost affects the cycle length (Proposition 4) as well as the follower’s profit rate (Section 6). Note that a lower fixed ordering cost, which always reduces the cycle length, is not always better for the follower.

There are several directions in which to extend this research. First, horizontal competition among manufacturers or among retailers can be taken into consideration, as well as competition among multiple supply chains. Second, the assumption of a deterministic demand function can be relaxed, and the replenishment policy can be adjusted according to a stochastic EOQ model (e.g., using a \((Q,R)\) model). A third direction is to consider scenarios in which the follower is the party that sets the cycle length, or in which the cycle length is a result of a bargaining process between the parties (e.g., Nash bargaining). Finally, a good avenue for further research would be to consider a revenue-sharing contract with a similar framework.
Appendix

Proof of Lemma 1.
(i) \( \frac{d^2\gamma}{dT^2} = \frac{d}{dT} \left( \frac{d\gamma}{dT} \right) = \frac{d}{dT} \left( \frac{Tg(T) - \int_0^T f(t)dt}{T} \right) = \frac{1}{T^2} \left( Tg(T) - \int_0^T f(t)dt \right) \leq \frac{1}{T^2} \left( Tg(T) - \int_0^T f(T)dt \right) = 0; \)
(ii) Using l'Hôpital's rule,
\[
\lim_{T \to 0^+} \frac{d\gamma}{dT} = \lim_{T \to 0^+} \frac{\int_0^T g(t)dt}{T} = \lim_{T \to 0^+} g(T) = g(0) = 1 \quad \text{and} \quad \lim_{T \to \infty} \frac{d\gamma}{dT} = \lim_{T \to \infty} \frac{\int_0^T g(T)dt}{T} = \lim_{T \to \infty} g(T) = 0.
\]
Following (i), \( I_I / T \) is a decreasing function of \( T \). The claim is proved by combining these two properties. (iii) Using l'Hôpital's rule, \( \lim_{\tau \to 0^+} \tau = \lim_{\tau \to 0^+} \frac{\int_0^\tau g(\tau)dt}{\tau} = \lim_{\tau \to 0^+} \frac{\tau g(\tau)}{\tau} = \lim_{\tau \to 0^+} g(\tau) = g(0) = 1 \), whereas the proof that \( \tau \) is an increasing function of \( T \) follows from Lemma 1 in Avinadav (2018).

Proof of Proposition 2
Using (10), the definition of \( I_I \) and \( I_F \), and according to (1) and (2), the retailer's and manufacturer's profits per cycle are, respectively,
\[
\Pi_R(T, \theta, w, m, s) = -(1 - \theta) T C(s) + (m - h_\gamma)f(p, s) I_I,
\]
\[
\Pi_M(T, \theta, w, m, s) = -\theta T C(s) + (w - c)f(p, s) I_I.
\]
Using (A.1) and (A.2), optimization problems (6) and (9), which refer to Model 1 and Model 4, respectively, can be written as follows:
\[
\max_{w,s} \left( (w - c)f(w + m(T, \theta, w), s(T, \theta, w)) I_I - \theta T C(s) \right)
\]
\[
S.t. \quad (m(T, \theta, w), s(T, \theta, w)) = \arg\max\left( (m - h_\gamma)f(w + m, s) I_I - (1 - \theta) T C(s) \right)
\]
and
\[
\max_{\theta, m} \left( (m - h_\gamma)f(w(T, \theta, m) + m, s(T, \theta, m)) I_I - (1 - \theta) T C(s(T, \theta, m)) \right)
\]
\[
S.t. \quad (w(T, \theta, m), s(T, \theta, m)) = \arg\max\left( (w - c)f(w + m, s) I_I - \theta T C(s) \right)
\]
\[
(A.3)
\]
\[
(A.4)
\]
We replace the decision variables of the models as follows: let \( \eta \) be the investment share of the leader and \( \eta_i, i \in \{L, F\} \), be net unit profit margin, where \( L \) denotes leader and \( F \) follower. For example, when the manufacturer is the leader (Model 1) \( u_L = w - c \) and \( u_F = m - h_\gamma \). In addition, a new parameter \( C_\gamma \equiv c + h_\gamma \) is introduced, denoting the average total unit cost (production plus holding). Consequently, both (A.3) and (A.4) can be formulated as
\[
\max_{\eta_i, \eta} \left( (u_i - c)f(u_i + u_F(T, \eta_i, u_F, c + C_\gamma, s(T, \eta_i, u_F)) I_I - \eta T C(s(T, \eta_i, u_F)) \right)
\]
\[
S.t. \quad (u_i(T, \eta_i, u_F), s(T, \eta_i, u_F)) = \arg\max_{\eta_i, \eta} \left( u_i f(u_i + u_F + C_\gamma, s) I_I - (1 - \eta) T C(s) \right)
\]
\[
(A.5)
\]
Thus, (A.5) represents a uniform formulation for the cooperative investment case regardless of the identity of the leader.

Using (A.1), (A.2) and the results of Proposition 1, optimization problems (7) and (8), which refer to Model 2 and Model 3, respectively, can be written as follows:
\[
\max_{w,s} \left( (w - c)f(w + m(T, w, s), s) I_I - T C(s) \right)
\]
\[
S.t. \quad m(T, w, s) = \arg\max_{w} \left( (m - h_\gamma)f(w + m, s) I_I \right)
\]
\[
(A.6)
\]
and
\[
\max_{m,s} \left( (m - h_\gamma)f(m + m(T, m, s), s) I_I - T C(s) \right)
\]
\[
S.t. \quad w(T, m, s) = \arg\max_{w} \left( (w - c)f(w + m, s) I_I \right)
\]
\[
(A.7)
\]
Both (A.6) and (A.7) can be formulated as
\[
\max_{u_L, \eta} \left( (u_L - c)f(u_L + u_F(T, u_L, c) + C_\gamma, s) I_I - T C(s) \right)
\]
\[
S.t. \quad u_L(T, u_L, c) = \arg\max_{u_L} \left( u_L f(u_L + u_F + C_\gamma, s) I_I \right)
\]
\[
(A.8)
\]
Thus, (A.8) represents a uniform formulation for the case of non-cooperative investment, regardless of the identity of the leader.

(i) Straightforward from (A.5) and (A.8);
(ii) Straightforward from (A.5) and (A.8), and since the conditional equilibrium selling price is equal to \( u_L + u_F + C_\gamma \).

Proof of Theorem 1

(i) Straightforward from (A.5) and (A.8);
(ii) Straightforward from (i).

Proof of Proposition 3.
Let \( N(T, K) \equiv \frac{\eta^T(T) - K}{T} \). Suppose \( K_1 < K_2 \), such that \( K_2 = K_1 + \Delta K \), \( \Delta K > 0 \). Suppose \( T_i \) satisfies
\[ \frac{d\pi(T, K_i)}{dT} = \frac{d}{dT} \left( \frac{\Pi^e_i(T)}{T} \right) + \frac{K_i}{T^2} = 0 \]  
(A.9)

and \( T_i \) satisfies
\[ \frac{d\pi(T, K_i)}{dT} \bigg|_{T=T_i} = \frac{d}{dT} \left( \frac{\Pi^e_i(T)}{T} \right) + \frac{K_i + \Delta K}{T^2} = 0. \]  
(A.10)

Thus, by substituting \( T_i \) in (A.10) and using (A.9), we obtain
\[ \frac{d\pi(T, K_i)}{dT} \bigg|_{T=T_i} = \frac{d}{dT} \left( \frac{\Pi^e_i(T)}{T} \right) + \frac{K_i}{T_i^2} + \frac{\Delta K}{T_i^2} > 0. \]  
(A.11)

Consequently, \( \eta_i(T, K_i) \) increases at \( T_i \), so \( T_i > T_i \).

Proof of Proposition 4.

By combining the results of Theorem 1(i) and Proposition 3.

Proof of the expressions in Table 2.

Model A1

Following (A.3) and using (12) and (14), the conditional equilibrium for a given \( T \) is obtained by solving the first-stage optimization problem

\[ \max_{E\omega} \left\{ (w - c)(a_i - a_0(w + T, \beta, w)) + s(T, \beta, w)I_T - \frac{d\gamma s(T, \beta, w)I_T^2}{2} \right\} \]

\[ S. \ t. \ (m(T, \beta, w), s(T, \beta, w)) = \text{argmax} \left\{ (m - h\eta)(a_i - a_0(w + m) + s)I_T - \frac{(1 - \beta)T^2}{2} \right\} \]  
(A.12)

We start by finding the retailer's best response from the constraint in (A.12). The Hessian of the retailer's profit function in the constraint is

\[ -2a_1 I_T I_T - (1 - \beta)TY \]

which is negative definite for

\[ 2a_1 \gamma (1 - \beta) > I_T > 0. \]  
(A.13)

Under this condition, the retailer's profit is maximized for \( s(T, \beta, w) = \frac{(m - a_0(w + h\eta))\phi_i}{\phi_i} \) and \( m(T, \beta, w) = \frac{\frac{1}{\beta} + a_0(w + h\eta) - a_0}{\phi_i} + h\eta \). Substituting these results in the objective function in (A.12), we find two extreme points that satisfy the necessary condition. One gives a negative profit for the manufacturer and zero profit for the retailer, and thus is omitted. The other, \( \beta = 1/3 \) and \( w = c + T(8a_1\gamma - 3\phi_i)\phi_i^{1/3}/a_0 \), satisfies the second-order condition (a negative definite Hessian of the manufacturer's profit function at the point) when:

\[ \gamma > \frac{9\phi_i}{16a_0} \]  
(A.14)

and thus is considered conditional equilibrium \((\beta^{A1}, w^{A1})\). In addition, we define \( m_i^{A1}, s_i^{A1} = m(T, \beta^{A1}, w^{A1}) \), \( s(T, \beta^{A1}, w^{A1}) \). The profits are calculated by substituting the conditional equilibrium in (A.1) and (A.2) combined with (12) and (14). Substituting \( \beta^{A1} = 1/3 \) in (A.13) yields

\[ \gamma > \frac{3\phi_i}{4a_0} \]  
(A.15)

Since the condition in (A.15) is stricter than that in (A.14), the latter is redundant. If (A.15) does not hold, then at least one party has an infinitely large profit since the Hessian of the retailer's profit is not negative definite for \( \gamma \leq \frac{3\phi_i}{4a_0} \), and the Hessian of the manufacturer's profit at conditional equilibrium is not negative definite for \( \gamma \leq \frac{9\phi_i}{16a_0} \).

Model A2

Following (A.6), (13) and (14), the conditional equilibrium for a given \( T \) is obtained by solving the first-stage optimization problem

\[ \max_{a_i, w} \left\{ (w - c)(a_i - a_0(w + m(T, s, w)) + s)I_T - \frac{2m^2}{2} \right\} \]

\[ S. \ t. \ m(T, s, w) = \text{argmax} \left\{ (m - h\eta)(a_i - a_0(w + m) + s)I_T \right\} \]  
(A.16)

We start by solving the constraint in (A.16). Since its argument is a concave parabolic function of \( m \), the retailer's best response is \( m(T, s, w) = \frac{2a_0(w + h\eta) + a_i - a_0}{a_i} \). Substituting this in the objective function of (A.16), we find a single extreme point \( (s, w) = (\phi_i, \psi_i^{A2}, c + 2\phi_i\psi_i^{A2}) \) which also satisfies the second-order condition (a negative definite Hessian of the manufacturer's profit) when

\[ \gamma > \frac{\phi_i}{4\phi_i} \]  
(A.17)

Thus, if (A.17) holds, then this extreme point is considered as conditional equilibrium \((s^{A2}, w^{A2})\). In addition, we define \( m^{A2} = m(T, w^{A2}, s^{A2}) \). The profits are calculated by substituting the conditional equilibrium \((w^{A2}, s^{A2}, m^{A2})\) in (A.1) and (A.2) combined with (12) and (14). If (A.17) does not hold, then the manufacturer has infinitely large profit since the Hessian of the manufacturer's profit is not negative definite for \( \gamma \leq \frac{3\phi_i}{4a_0} \).

Model A3

Similar to the proof of Model A2.

Model A4

Similar to the proof of Model A1.


(i) Straightforward from Table 2 and since by Lemma 1(iii), \( \Lambda_T \) is a decreasing function of \( T \). (ii) \( \frac{\phi_i^{A1}}{\phi_i^{A2}} = \frac{(a - \psi_i^{A2}/(\psi_i^{A2}))}{a - \psi_i^{A2}/(\psi_i^{A2})} = \frac{6(4 - \phi_i/\psi_i^{A2})}{6 - \phi_i/\psi_i^{A2}} = \frac{6(4 - \phi_i/\psi_i^{A2})}{6 - \phi_i/\psi_i^{A2}} \) is a
monotone increasing function of $\phi_2/(\alpha_3\gamma)$. Since, by Lemma 1, $\phi_2 > 0$, then $s_{[2]}^M/s_{[2]}^M \geq 1.5$.

Proof of Observation A2.

\[
\frac{p_{[2]}^M - c - h_{T,2}}{p_{[2]}^M - c - h_{T,1}} = \frac{(4q_{[2]}/\phi_2 - 1)^2}{4q_{[2]}/\phi_2 - 1 - \phi_2/\phi_2} = \left(1 - \frac{1}{4q_{[2]}/\phi_2}\right)\left(1 + \frac{5}{4q_{[2]}/\phi_2}\right),
\]
which is a decreasing function of $\alpha_3\gamma/\phi_2$ for $\alpha_3\gamma/\phi_2 > 3/4$. Therefore, since
\[
\lim_{\alpha_3\gamma/\phi_2 \to \infty} \frac{p_{[2]}^M - c - h_{T,2}}{p_{[2]}^M - c - h_{T,1}} = 1,
\]
then $\frac{p_{[2]}^M - c - h_{T,2}}{p_{[2]}^M - c - h_{T,1}} > 1$. By Proposition 2(ii), we can infer that $\frac{p_{[2]}^M - c - h_{T,2}}{p_{[2]}^M - c - h_{T,1}} > 1$, which is supported by the results in Table 2.

Proof of Observation A3.

\[
\frac{w_{[2]}^M}{w_{[2]}^M} = \frac{4q_{[2]}/\phi_2 - 1 - \phi_2/\phi_2}{4q_{[2]}/\phi_2 - 1} = 1 + \frac{5}{4q_{[2]}/\phi_2 - 1}.
\]
According to the first row in Table 2, $\alpha_3\gamma/\phi_2 > 3/4$ is required to ensure the existence of conditional equilibrium, which proves the claim $1 < \frac{p_{[2]}^M - c - h_{T,2}}{p_{[2]}^M - c - h_{T,1}} < 2^2$.

Proof of Observation A5.

(i) By Table 2, $\frac{w_{[2]}^M}{w_{[2]}^M} = 1 + \frac{1}{4q_{[2]}/\phi_2 - 1}$. Thus, similarly to the proof of Observation A2, 1 < $\frac{p_{[2]}^M(T)/\Pi_{[2]}^M(T)}{p_{[2]}^M(T)/\Pi_{[2]}^M(T)} < 2^2$. By Theorem 1(i), we can infer that 1 < $\Pi_{[2]}^M(T)/\Pi_{[2]}^M(T) < 2^2$, which is supported by the results in Table 2.

(ii) $\frac{\Pi_{[2]}^M(T)}{\Pi_{[2]}^M(T)} = \frac{4q_{[2]}/\phi_2 - 1 - \phi_2/\phi_2}{4q_{[2]}/\phi_2 - 1} = \left(1 + \frac{1}{4q_{[2]}/\phi_2 - 1}\right)^2$, which is a monotone decreasing function for $0 < \phi_2/(\alpha_3\gamma) < 4/3$, which proves the claim 0 < $\Pi_{[2]}^M(T)/\Pi_{[2]}^M(T) < 1$. By Theorem 1(i), we can infer that 0 < $\Pi_{[2]}^M(T)/\Pi_{[2]}^M(T) < 1$, which is supported by the results in Table 2.

Proof of Observation A6.

\[
\frac{\Pi_{[2]}^M(T)}{\Pi_{[2]}^M(T)} = \left(1 - \frac{11}{4q_{[2]}/\phi_2 - 1}\right)\left(1 + \frac{5}{4q_{[2]}/\phi_2 - 1}\right),
\]
which is a decreasing function of $\alpha_3\gamma/\phi_2$ for $\alpha_3\gamma/\phi_2 > 3/4$. Therefore, since
\[
\lim_{\alpha_3\gamma/\phi_2 \to \infty} \frac{\Pi_{[2]}^M(T)}{\Pi_{[2]}^M(T)} = 1,
\]
then $\frac{\Pi_{[2]}^M(T)}{\Pi_{[2]}^M(T)} > 1$. By Theorem 1(ii), we can infer that $\frac{\Pi_{[2]}^M(T)}{\Pi_{[2]}^M(T)} > 1$, which is supported by the results in Table 2.

Proof of Observation A7.

Straightforward from Observations A5.

Proof of the expressions in Table 3.

Model M1.

Following (A.4) and using (13) and (14), the conditional equilibrium for a given $T$ is obtained by solving the first-stage optimization problem, which, due to the multiplicative demand function, can be written as

\[
\begin{align*}
\max_{\delta,w} & \left\{w + (m(T, w) - h_{[2]}\gamma/\phi_2)(a_2 - \alpha_3\gamma)(m(T, w) + w)\right\} \\
\text{s. t.} & \quad (m(T, w) - h_{[2]}\gamma/\phi_2)(a_2 - \alpha_3\gamma)(m(T, w) + w)\right\} \\
& \quad m(T, w) = \text{argmax}(m - h_{[2]}\gamma/\phi_2)(a_2 - \alpha_3\gamma)(m(T, w) + w))
\end{align*}
\]

(A.18)

We start by solving the second constraint in (A.18). Since its argument is a concave parabolic function of $m$, the retailer’s best response is $m(T, w) = \frac{a_2 - \alpha_3\gamma(w + m(T, w))}{2}$. Since the argument of the first constraint in (A.18) is a concave parabolic function of $s$, the retailer’s best response is $s(T, \delta, w) = \frac{2a_2 - 2\alpha_3\gamma(w + h_{[2]}\gamma/\phi_2)}{4a_2 - 2\alpha_3\gamma(2)\gamma/\phi_2}$. Substituting these results in the objective function of (A.18), we find the conditional equilibrium $(\delta_{[2]}^M, w_{[2]}^M)$. In addition, we define $m_{[2]}^M = (m(T, \delta_{[2]}^M, w_{[2]}^M), s(T, \delta_{[2]}^M, w_{[2]}^M))$. The profits are calculated by substituting the conditional equilibrium in (A.1) and (A.2) combined with (13) and (14).

Model M2.

Following (A.7), (13), (14) and the result of Proposition 3, the conditional equilibrium for a given $T$ is obtained by solving the first-stage optimization problem

\[
\begin{align*}
\max_{\delta, w} & \left\{w + (m(T, w) - h_{[2]}\gamma/\phi_2)(a_2 - \alpha_3\gamma)(m(T, w) + w)\right\} \\
\text{s. t.} & \quad (m(T, w) - h_{[2]}\gamma/\phi_2)(a_2 - \alpha_3\gamma)(m(T, w) + w)\right\} \\
& \quad m(T, w) = \text{argmax}(m - h_{[2]}\gamma/\phi_2)(a_2 - \alpha_3\gamma)(m(T, w) + w))
\end{align*}
\]

(A.19)

We start by solving the constraint in (A.19). Since its argument is a concave parabolic function of $m$, the retailer’s best response is $m(T, w) = \frac{a_2 - \alpha_3\gamma(w + m(T, w))}{2}$. Substituting this result in the objective function of (A.19), we find the conditional equilibrium $(w_{[2]}^M, s_{[2]}^M)$. In addition, we define $m_{[2]}^M = (m(T, w_{[2]}^M))^2$. The profits are calculated by substituting the conditional equilibrium in (A.1) and (A.2) combined with (13) and (14).

Model M3.

Similar to the proof of Model M2.

Model M4.

Similar to the proof of Model M1.

Proof of Observation M7.

Straightforward from Observations M2 and M5.

References


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