A risk-averse and buyer-led supply chain under option contract: CVaR minimization and channel coordination

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\begin{abstract}
Option contracts are used commonly in business to tackle difficulties in working capital shortage and hedge market risks. In this paper, we consider option contract application in a buyer-led supply chain, where both the buyer and supplier are risk-averse. The effects of option price and option exercise price are investigated via conditional value-at-risk (CVaR) minimization. A Stackelberg game model is established to examine the influences of adjusting both prices on the benefits and risks at the buyer and supplier sides. We find that the increase of both prices, especially the increase of option price, benefits the supplier but causes loss to the buyer. The supply chain's total risk is not affected by either price when the buyer and supplier have the same risk preference. However, when the supplier raises the option price, the buyer who is more risk-averse will bear extra risk more than the supplier's reduced risk. A numerical study shows that the option exercise price has an opposite effect on the supply chain. We theoretically prove that when the supplier's risk preference is the same as or larger than that of the buyer, the supply chain can be coordinated under option contract; otherwise, the supply chain cannot be coordinated.
\end{abstract}

1. Introduction

Supply chain finance (SCF) is regarded as an effective means to hedge risk and optimize the financial structure and cash-flow at an inter-organizational level within the supply chain (Gomm, 2010). The implementation of SCF has attracted close attention from the industry and government. For example, the UK Prime Minister announced a Supply Chain Finance Scheme in 2012 (Gelsomino et al., 2016) and China's State Council released a plan to advance the development of SCF in 2017.\textsuperscript{1} Accordingly, the research on SCF has grown significantly in the last decade. Gelsomino et al. (2016) identify two major perspectives in the extant literature: the finance-oriented perspective which focuses on financial solutions provided by financial institutions, and the supply chain-oriented perspective which focuses on working capital optimization. The second perspective emphasizes the role of coordination among supply chain partners in mitigating financial risk, reducing working capital, obtaining savings, and creating profits (Soufani, 2001; Gelsomino et al., 2016; Pellegrino et al., 2018).

Option is usually regarded as a high-leverage financial tool, since the buyer gets the right of purchasing from option contracts by investing only a small amount of money and finally may obtain quite substantial profits after exercising them. For risk hedging, when the market does not thrive, the option offers the buyer an opportunity to avoid over-purchasing loss, as the buyer has the right not to exercise the option in the future. As a result, what the buyer would lose is only the money that he has paid for option purchasing. From another perspective, the buyer who is in short of working capital can also apply the option contract which costs less money than the fixed committed order at present to guarantee future supplies of products. Therefore, option contract is increasingly popular in supply chain finance, and is regarded as an effective tool to hedge price and demand risks, as well as reduce costs (Wu and Kleindorfer, 2005).

Many studies show that option contract ameliorates the buyer’s situation (Donohue, 2000; Wu and Kleindorfer, 2005; Cai et al., 2017). By contrast, the supplier, who is usually the option writer, has to bear the risk shifted from the buyer and may suffer huge losses (Jörnsten et al., 2012). However, there are few studies from the supplier's perspective and hence further studies are required to investigate the risks at the supplier side and at the supply chain level in option contracts. A global investigation of executives demonstrates that “they were

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only willing to accept a risk of loss from 1 to 20 percent ... even when the size of the investment was smaller by a factor of ten" (Koller et al., 2012, p. 15–17). It is evident that most executives are risk averse. A method to measure the downside risk is value-at-risk (VaR). However, it provides no handle on the extent of the losses that might be suffered beyond the threshold amount indicated by the VaR measure. An alternative measure that quantifies the losses that might be encountered in the tail is conditional value-at-risk (CVaR), which has been proved coherent, stable and convex (Rockafellar and Uryasev, 2002), and has emerged as a practical approach for modelling risk aversion with wide applications in economics, finance and insurance (Rockafellar and Uryasev, 2000).

An increasing number of studies have focused on the application of option contracts to procurement risk management (Wu and Kleindorfer, 2005; Wang et al., 2012; Liu et al., 2014; Anderson et al., 2017; Zhao et al., 2018). However, rare research has examined how risk as an exogenous factor affects the supply chain decisions involved in option contracts (Zhuo et al., 2018). This motivates us to investigate a supply chain consisting of both risk-averse buyer and supplier under option contracts facing market uncertainty. Specifically, this paper tries to analyze the influence of option contract on risk in a one-buyer-one-supplier supply chain model, and the buyer takes the lead in a Stackelberg game. To the best of our knowledge, there is rare study considering a risk-averse supply chain incorporating option order and committed order simultaneously under CVaR criterion. This paper makes three major contributions as follows.

1. **Option-related prices:** This paper theoretically proves that if a supplier has the ability to price an option, the supplier will increase option price rather than option exercise price to reduce risk, which in turn causes the buyer to bear more risk. It implies that a good supply chain finance strategy for the risk averse supplier is to work with a less risk averse buyer and supply chain risk management is important for both parties.

2. **Risk preference of decision makers:** We analytically find that the influence of option pricing on the supply chain's total risk is closely related with the buyer's and the supplier's risk preference. When they have the same risk preference, the option price and the option exercise price will not affect the supply chain's total risk. Furthermore, if the buyer is more risk averse, the increase of the option price will bring extra risk at the buyer side larger than the supplier's reduced risk, which means that the total supply chain risk will be increased. If a supplier is more risk-averse, increasing the option price will decrease the risk transferred from the buyer, which indicates a reduced total supply chain risk. In addition, we prove that a higher buyer's risk aversion level leads to more loss for the supplier. The results further support the supplier's strategy of working with a less risk averse buyer, which will also reduce the total supply chain risk.

3. **Supply chain coordination:** We theoretically prove that a supply chain can be coordinated under option contract via CVaR minimization and find that the supply chain cannot be coordinated if the buyer is more risk averse than the supplier. The results provide guides for the buyer and supplier to minimize the supply chain risk.

The rest of this paper is organized as follows. In the next section, we review the relevant literature. Section 3 presents a brief description of the one-buyer-one-supplier, buyer leading Stackelberg model and some preliminaries on CVaR measure. Section 4 investigates the optimal strategy for the buyer and supplier under CVaR minimization. Supply chain coordination under CVaR criterion is analyzed in Section 5. Section 6 presents the numerical studies, and Section 7 concludes this paper and discusses the future research.

**2. Literature review**

In this section, we review two research streams in the extant literature, i.e. option contracts in the supply chain, and risk measure and analysis including CVaR.

For the research stream of option contracts, the objective of most studies is to maximize the expected profits or minimize the expected costs. Wu and Kleindorfer (2005) develop a framework to analyze business-to-business transactions and supply chain management based on integrating contract procurement markets with spot markets using capacity options and forwards. Wang and Tsao (2006) build a single-period two-stage supply contract, from which the buyer can adjust the initial order and then they formulate the buyer's optimal policies. Feng et al. (2014) investigate a purchasing contract with options under capital constraint and credit support, where through a marginal comparison, the buyer's optimal ordering strategy is characterized. Liu et al. (2014) study the option contract and advanced purchasing discount contract and present the optimal decisions at both the retailer and manufacturer's aspects.

Most investigations of coordination are based on profit maximization or cost minimization. Donohue (2000) regards options to transfer the risk from a supplier to a buyer in a two-stage model, where return policies are employed to achieve supply chain coordination. Barnes-Schuster et al. (2002) analyze the role of options in a two-period correlated demand model and find that supply chain coordination can be achieved only if the exercise price is piecewise linear. Wang et al. (2007) consider a dominant retailer who aims to coordinate the upstream production quantity and develop a model to study channel coordination and risk sharing through option contract. Their results show that the proposed contract improves the supply chain's profit. Chen and Shen (2012) study a one-period two-party supply chain with a service requirement and show that a special class of contracts can coordinate the supply chain with option contracts. Chen et al. (2017) further investigate the bidirectional option contracts with service requirement, and find these contracts are beneficial to both the retailer and the supplier. Apart from the market uncertainty, Cai et al. (2017) introduce an option contract to improve the performance of a vendor-managed inventory (VMI) supply chain under yield uncertainty.

Stackelberg game and its extensions are widely applied in the research of option contract. Zhao et al. (2010) take a cooperative game approach to consider the coordination issue in a manufacturer-retailer supply chain using option contract. They find that unlike the wholesale price mechanism, option contract coordinates the supply chain and achieves Pareto improvement. Zhao et al. (2013) take the value-based approach to price the real supply chain options and show that their pricing strategies are more objective and fairer than the results of traditional Stackelberg game approach. Arani et al. (2016) consider a mixed revenue-sharing option contract to coordinate a retailer-manufacturer supply chain, where the proposed mixed contract is modeled by a game theoretic approach to obtain the retailer's order quantity and manufacturer's production quantity. However, to the best of our knowledge, few studies have taken into consideration the option contract analysis under CVaR criterion.

For the research stream in risk measure and analysis, prior studies pay attention to the risk analysis of option contracts. For example, Lau and Lau (1999) discuss how a manufacturer could establish her pricing and return policy by properly considering not only the production cost and retail sale price, but also her risk attitude and that of the retailer, as well as the demand uncertainty. Chen and Parlar (2007) investigate an extension of single-period inventory model. They find that the option strike price and strike quantity do not affect the maximum profit but the variance of the profit. Choi et al. (2008) propose to use a mean-variance formulation to quantify risk proneness and risk aversion in the supply chain. They study how the manufacturer can set the wholesale price to achieve channel coordination in a buyback contract. Jörnsten et al. (2012) study the transfer of risk in the newsvendor model with discrete
demand by analyzing expected profit and variance. They find that the real option contract in some cases can offer an improved position for the retailer. Feng and Wu (2018) examine an option contract from a supplier’s perspective and apply mean-variance method to analyze the supplier’s risk. They find that the option contract can also benefit the supplier. Zhuo et al. (2018) study the implications of risk considerations for option contracts in a two-echelon supply chain under the mean–variance framework. According to Emmer et al. (2015), the adoption of variance assumes that distributions of risks are approximately symmetric around the mean since the variance does not distinguish between positive and negative deviations from the mean. By contrast, CVaR is a coherent risk measure given by a “worst case method” (Artzner et al., 1999; Rockafellar and Uryasev, 2002), which differentiates our research from Zhuo et al. (2018).

The research stream of CVaR measure starts from Rockafellar and Uryasev (2000, 2002), in which CVaR and its minimization formula are first proposed. Following the fundamental properties of CVaR, the single-product newsvendor model with the risk preferences of the decision maker expressed by CVaR has been examined by many scholars. Chen et al. (2004) firstly introduce the CVaR measure into the study of the newsvendor model and prove that without shortage cost consideration the risk-averse newsvendor’s optimal order quantity is smaller than that under expected profit maximization. Gotoh and Takano (2007) show that the CVaR is tractable in the newsvendor problem and provide analytical solutions with two different loss functions of net loss and total cost. Jammernegg and Kischka (2007) consider two risk parameters of the confidence level and the coefficient of pessimism and propose a mean-CVaR newsvendor model under CVaR criterion. They obtain the optimal decision of a risk sensitive newsvendor and analyze how the decision is affected by the risk attitude. This mean-CVaR model is also applied in Xie et al. (2016). Based on the retailer’s wholesale price contracts, buy-back contracts and revenue-sharing contracts, they investigate how a supply chain can be coordinated under the mean-CVaR criterion.

A variety of extensions of the newsvendor model under CVaR criterion have been investigated after Gotoh and Takano (2007) and Jammernegg and Kischka (2007). Zhou et al. (2008) propose a return-CVaR model with stochastic market demand, which provides the optimal decision-making approach for decision makers with multi-product ordering quantity problem. Yang et al. (2009) study the coordination of supply chains with a risk-neutral supplier and a risk-averse retailer under CVaR criterion and show that the supply chain can be coordinated with the revenue sharing, buy-back, two-part tariff and quantity flexibility contracts. Chen et al. (2009) consider the risk-averse newsvendor’s optimal pricing and inventory decisions in additive and multiplicative demand models, respectively, and characterize the monotoncity properties of the models by performing comparative statics. Wu et al. (2013) study the effect of capacity uncertainty on a risk-averse newsvendor’s inventory decisions with considering VaR and CVaR, where they find that risk-neutral newsvendor’s optimal order quantity is not affected by the capacity uncertainty but the risk-averse one is. Wu et al. (2014) investigate a competitive newsvendor problem under CVaR criterion, where for quantity competition, they consider proportional demand allocation and demand reallocation, and for price competition they consider the situation of both additive and multiplicative demand. Chen et al. (2014) investigate the impact of a target on newsvendor decisions and model the effect of a target by maximizing the satisfying measure (e.g. CVaR) of a newsvendor’s profit with respect to that target. Xu et al. (2015) introduce loss aversion in the newsvendor decision bias under CVaR and propose a new order policy to obtain the optimal order quantities for different objective functions.

In the extant literature, there is a dearth of application of CVaR measure in option contract analysis. Yang et al. (2009) and Xie et al. (2016) discuss the buy-back contract on supply chain coordination, which is a special case of option contract (Barnes-Schuster et al., 2002). Both studies are based on the newsvendor problem and only analyze the wholesale price of committed order, which is just a part of option pricing. A recent closely related study is Xue et al. (2015), which considers the option contract and CVaR measure simultaneously. They derive structural results on the optimal ordering and put option decisions as well as how the system parameters, risk-averse attitude and demand uncertainty affect the value of option. However, their work only considers the options, does not incorporate the committed order at the same time; and they do not answer the question of how option-relates prices (option prices and option exercise price) influence risk. Since the price of committed order is lower than that of option order, it helps to incorporate committed order in the option contract in order to reduce the buyer’s loss and increase the supplier’s revenue, and deserves further investigation as a powerful supply chain finance strategy. Therefore, this study considers both committed order under wholesale price and option order under option contract with CVaR measure, and hence analyzes the risk shifting between the buyer and supplier and supply chain risk coordination, thereby contributing to the relevant literature.

3. Model description, notations and preliminaries

3.1. Model description and notations

This paper focuses on a one-buyer-one-supplier supply chain, which runs over a period from 0 to 1. At time 0, the buyer decides to order Q units at the wholesale price w per unit. We refer to Q as the committed order, which will be delivered at time 1, the beginning of the selling season. Meanwhile, the buyer decides to purchase M commodity options at the unit option price w. Given that market demand is observed at time 1, the buyer may exercise options m that are less than M units (m ≤ M) at the unit exercise price of w. We assume that one option gives the buyer the right to purchase one unit of goods. Accordingly, the supplier benefits from the committed order quantity Q at the unit wholesale price w and the option order M at the unit option price w at time 0, and the option exercise quantity m at the unit exercise price w at time 1. From time 0 to 1, the retailer sells the goods to the market at unit price p. f(x) is the probability density function of the market demand estimated at time 0, and F(x) is its cumulative distribution function. Both functions are assumed to be continuous and F(x) is assumed to be differentiable.

The buyer and supplier’s decisions are made at time 0, when the quantities of committed order and option order and their prices are confirmed. In a buyer-led supply chain, the buyer decides the order quantity and the supplier accepts the order from the buyer. Let’s consider a Stackelberg game of the buyer-led supply chain. In the first stage, the buyer minimizes her CVaR by confirming the committed order quantity and option order quantity. In the second stage, under the buyer’s optimal order quantity, the supplier minimizes her CVaR through pricing option contract properly.

The buyer decides the quantities of committed order and option order at time 0. The buyer’s profit function πb(Q, M, x) is described as:

\[
\pi_b(Q, M, x) = \begin{cases} 
\pi_1 = px - wQ - w_0M, & x \leq Q \\
\pi_2 = px - w(x - Q) - wQ - w_0M, & Q < x \leq Q + M \\
\pi_3 = p(Q + M) - wQ - w_0M - wM, & x > Q + M 
\end{cases}
\]

(1)

Correspondingly, the supplier’s profit function πs(Q, M, x) is:

\[
\pi_s(Q, M, x) = \begin{cases} 
\pi_4 = wQ + w_0M - c(Q + M), & x \leq Q \\
\pi_5 = w(x - Q) + wQ + w_0M - c(Q + M), & Q < x \leq Q + M \\
\pi_6 = w_0M + wQ + w_0M - c(Q + M), & x > Q + M 
\end{cases}
\]

(2)

In Equations (1) and (2), wQ + w_0M is the supplier’s revenue or the...
buyer’s cost of trading the committed order \( Q \) and option order \( M \), and 
\( c(Q + M) \) is the supplier’s production cost. When \( x \leq Q \), the buyer’s profit is the sales income of \( px \) minus the cost of \( wQ + w_0M \) paid for the committed order and option order and the supplier obtains the profit of \( \pi_a \). When \( Q < x \leq Q + M \), the buyer will exercise \( Q \) units of options with the cost of \( w_0(x - Q) \) and the supplier has the extra profit of 
\( w_0(x - Q) \). When \( x > Q + M \), the buyer will exercise all the purchased options of \( M \) units with the cost of \( w_vM \) and the supplier’s additional profit from that is \( w_vM \).

An effective method to measure the cost is the net loss, or profit loss. Similar to the newsvendor model in Gotoh and Takano (2007), we define the net loss of the buyer and supplier as:

\[
L_0(Q, M, x) = -\pi_a(Q, M, x)
\]

and

\[
L_0(Q, M, x) = -\pi_a(Q, M, x)
\]

Meanwhile, for different situations, \( L_{d1} = -\pi_a \), \( L_{d2} = -\pi_a \), and \( L_{d3} = -\pi_a \) are defined for the buyer’s loss and \( L_{s1} = -\pi_a \), \( L_{s2} = -\pi_a \), and \( L_{s3} = -\pi_a \) are defined for the supplier’s loss. For ease of reference, the main model notations are summarized in Table 1.

We also have the following assumptions:

3.1.1. Assumptions

(a) \( 0 < w < p; \)
(b) \( 0 < w_0 + w_i < p; \)
(c) \( w < w_0 + w_i; \)
(d) \( w_0 < w; \)
(e) \( 0 < \varepsilon < w; \)

Assumptions (a) (c) and (e) are intuitive. Assumption (b) of \( w_0 + w_i < p \) describes that buying option can make profit. If \( w \leq w_0 \), options will become meaningless for the buyer; thus Assumption (d) is reasonable. Based on these assumptions, we have the following relationships of loss functions under different market demands.

Lemma 1. (a) \( L_{d1} \geq L_{d2} \geq L_{d3} \); (b) \( L_{s1} \geq L_{s2} \geq L_{s3} \).

Proof. In Appendix.

This lemma indicates that the lower demand from consumers will always bring higher cost to both the buyer and supplier after the buyer makes her decision.

3.2. Preliminaries about CVaR

CVaR measure is to investigate the downside risk which concentrates on the loss above a target level or the profit below a target level. Rockafellar and Uryasev (2000, 2002) have conducted well-rounded investigation about the properties of CVaR and we reintroduce some of them here, which are significant for our following analysis. For the problem to minimize the loss, with a random variable \( x \), which stands for the uncertainties, e.g. market demand, we assume \( \delta(q, x) \) is the decision maker’s loss from a decision vector \( q \) and \( \alpha \) is the threshold of loss. Since CVaR is the extension of VaR, we introduce VaR firstly, which is denoted by \( \alpha(q) \) and presented as:

\[
\alpha(q) = \min \{ \alpha \in \mathbb{R} | P(\delta(q, x) \leq \alpha) \geq \beta \}
\]

where \( P(\delta(q, x) \leq \alpha) \) represents the probability of \( \delta(q, x) \) under value \( \alpha \). Here, \( \beta \) denotes the confidence level, where \( \beta \in (0,1] \) reflects the degree of risk-aversion of the decision maker. The larger \( \beta \) is, the more risk-averse the decision maker is. \( \alpha(q) \) is the decision maker’s minimum loss under confidence level \( \beta \).

Based on VaR, CVaR is presented as the \( \alpha(q) \) as the targeted loss and denoted by \( \Phi \) as:

\[
\Phi(q) = \Phi(q) = E[\delta(q, x)|\delta(q, x) \geq \alpha(q)]
\]

As presented in the above function, \( \Phi(q) \) is the expected value of the loss exceeding the target level \( \alpha(q) \). To compute \( \Phi(q) \), Rockafellar and Uryasev (2000) introduce the following auxiliary function:

\[
G_\alpha(q, \alpha) = \alpha + \frac{1}{1 - \beta}E[(\delta(q, x) - \alpha)_+]
\]

and have proved the relationship of \( \alpha(q) \), \( \Phi(q) \) and \( G_\alpha(q, \alpha) \) presented in the following lemma.

Lemma 2.

(a) \( \alpha(q) = \arg \min G_\alpha(q, \alpha), \quad \Phi(q) = G_\alpha(q, \alpha) \) \quad (Theorem 10, Rockafellar and Uryasev, 2002);
(b) If \( \delta(q, x) \) is convex with respect to \( q \), then \( \Phi(q) \) is convex with respect to \( q \) as well. Indeed, in this case \( G_\alpha(q, \alpha) \) is jointly convex with respect to \( q, \alpha \) \quad (Corollary 11, Rockafellar and Uryasev, 2002);
(c) \( \min \Phi(q) = \min G_\alpha(q, \alpha) \) \quad (Theorem 14, Rockafellar and Uryasev, 2002).

4. CVaR minimization

In this section, we solve the Stackelberg game by minimizing the buyer and supplier’s CVaR and then analyze their decisions.

4.1. The Buyer’s decisions

In the first stage of the Stackelberg game, the buyer decides her optimal order quantities by minimizing her CVaR. The decisions of the buyer are the committed order quantity \( Q \) and the option order quantity \( M \). Based on (5), the buyer’s auxiliary function of CVaR is:

\[
R_b(Q, M, \alpha) = \alpha + \frac{1}{1 - \beta} \int_0^\infty [(\delta(q, M, x) - \alpha)_+]f(x)dx
\]

(6)

Lemma 2(c) indicates that minimizing the buyer’s CVaR is equivalent to minimizing the auxiliary function \( R_b(Q, M, \alpha) \), thus the minimization problem becomes:

\[
\min_{Q \geq 0, M \geq 0} R_b(Q, M, \alpha)
\]

(7)
and its solution is shown in Theorem 1. Hereafter, the buyer’s CVaR is denoted by $\Phi_b$.

**Theorem 1.**

(a) The optimal threshold of loss to minimize problem (7) is

$$\alpha_b = w_c M + w - w_Q - (p - w_e) F^{-1}(1 - \beta_b)$$

and is subject to

$$Q + M > F^{-1}(1 - \beta_b) > Q$$

(b) The buyer’s CVaR $\Phi_b$ is:

$$\Phi_b(Q, M) = w_Q + w - w_Q - (p - w_e) F^{-1}(1 - \beta_b) + \frac{p}{1 - \beta_b} \int_0^Q F(x)dx + \frac{p - w_e}{1 - \beta_b} \int_Q^\infty F(x)dx$$

(c) The optimal order quantities $(Q_b, M_b)$ to minimize $\Phi_b(Q, M)$ is:

$$\begin{cases} Q_b = F^{-1}(1 - \beta_b(w_c + w - w_e)) \\ M_b = F^{-1}(1 - \beta_b) - Q_b \end{cases}$$

**Proof.** In Appendix.

In Theorem 1, it is observed that $\alpha_b$ increases with confidence level $\beta_b$. The risk that the buyer will take is measured by $\Phi_b(Q, M)$. Obviously, the buyer’s loss mainly comes from her initial payment $w_Q + w_c M$ to the supplier. Besides, the final market price and option exercise price also have impact on the risk of loss.

**Corollary 1.**

(a) $Q_b$ is a decreasing function of wholesale price $w$ but an increasing function of option exercise price $w_e$ and option price $w_e$;

(b) $M_b$ is an increasing function of wholesale price $w$ but a decreasing function of option exercise price $w_e$ and option price $w_e$;

(c) Both $Q_b$ and $M_b$ are decreasing functions of confidence level $\beta_b$;

(d) $0 < \partial Q_b / \partial w < \partial Q_b / \partial w_e, \partial M_b / \partial w < \partial M_b / \partial w_e < 0$.

**Proof.** In Appendix.

Increasing option price or option exercise price means the wholesale price $w$ becomes relatively lower; thus, it is rational for the buyer to purchase more committed order, which is the result of Corollary 1(a), and reducing option order is also a choice which is the result of Corollary 1(b). As indicated in Lemma 1, low market demand is the critical risk for causing loss for the buyer; thus, this risk-averse buyer prefers both less committed order and option order in order to prevent the loss. Corollary 1(d) indicates that increasing option price has stronger influence than the option exercise price to let the buyer consider more committed order. This is because option price needs to be paid at the beginning of the option contract, but the option exercise price only needs to be paid when the buyer decides to exercise the option.

Since the option-related prices ($w_c, w_e$) are decided by the supplier, how the buyer is influenced by option contract pricing is valuable to be investigated. We have the following result.

**Proposition 1.** $0 < \partial \Phi_b(w_c, w_e) / \partial w < \partial \Phi_b(w_c, w_e) / \partial w_e$.

**Proof.** In Appendix.

From Proposition 1, the increase of option price and option exercise price results in the rise of the buyer’s CVaR. Additionally, the option price causes more risks compared with the option exercise price. This result is consistent with our usual understanding and the explanation is similar to Corollary 1(d). The increase in the price of options means that the buyer will spend more at the beginning. However, if the price of option exercise increases, in case the buyer does not exercise options afterwards, the losses will not occur.

### 4.2. The Supplier’s decisions

In the second stage of the Stackelberg game, after observing the buyer’s order quantities, the supplier decides the option price and option exercise price by minimizing her CVaR. Similar to the analysis of the buyer’s decision, based on the auxiliary function (5) and Lemma 2 (c), the supplier’s auxiliary function of CVaR is:

$$R_s(Q, M, \alpha) = \alpha + \frac{1}{1 - \beta_s} \int_0^\infty \left[ L_s(Q, M, x) - \alpha \right] f(x) dx$$

Minimizing the supplier’s CVaR is equivalent to minimize her auxiliary function $R_s(Q, M, \alpha)$. Thus, the minimization problem is:

$$\min_{Q \geq 0, M \geq 0, \alpha \in \mathbb{R}} R_s(Q, M, \alpha)$$

and its solution is shown in Proposition 2. Hereafter, the supplier’s CVaR is denoted by $\Phi_s$.

**Proposition 2.**

(a) The optimal threshold of loss to minimize problem (13) is:

$$\alpha_s = c(Q + M) - w_Q - w_c M + w_Q F^{-1}(1 - \beta_s)$$

which is subject to

$$Q + M > F^{-1}(1 - \beta_s) > Q$$

(b) The supplier’s CVaR $\Phi_s$ is:

$$\Phi_s(Q, M) = c(Q + M) - w_Q - w_c M + w_Q F^{-1}(1 - \beta_s) + \frac{w_c}{1 - \beta_s} \int_0^Q F(x)dx$$

(c) Let $B = \int_0^\infty F(x)dx/[F^{-1}(1 - \beta_s)(1 - \beta_s)]$, if $w_e < c$ and $(w_c + w_e - w)/w_e > B$, the optimal order quantities $(Q, M)$ to minimize $\Phi_s(Q, M)$ are:

$$\begin{cases} Q_s = 0 \\ M_s = F^{-1}(1 - \beta_s) \end{cases}$$

If $w_e < c$ and $(w_c + w_e - w)/w_e \leq B$,

$$\begin{cases} Q_s = F^{-1}(1 - \beta_s) \\ M_s = 0 \end{cases}$$

If $w_e \geq c$ and $(c + w_e - w)/w_e > B$,

$$\begin{cases} Q_s = 0 \\ M_s = \text{finite} \end{cases}$$

If $w_e \geq c$, and $(c + w_e - w)/w_e \leq B$,

$$\begin{cases} Q_s = F^{-1}(1 - \beta_s) \\ M_s = \text{finite} \end{cases}$$

**Proof.** In Appendix.

In (14), $\alpha_s$ shows the relationship of the supplier’s threshold of loss, confidence level and order quantity. The supplier’s optimal threshold of loss $\alpha_s$ increases with her confidence level $\beta_s$. Obviously, from the expression of $\Phi_s(Q, M)$, unlike the buyer, the supplier’s risk mainly comes from the production cost. Proposition 2 also shows that when the production cost is lower than option price, the option contract for the supplier is risk-free. This intuitive result helps to further refine the pricing range of options.

In the second stage of the Stackelberg game, the supplier decides her option price and option exercise price by minimizing her CVaR. With the buyer’s optimal order quantities (11), from (16) we can obtain the supplier’s minimization problem as:
\[
\min_{w, w_0} \Phi_1(w, w_0) \tag{21}
\]

We have the following result.

**Proposition 3.**

(a) \(0 > \delta \Phi_1(w_0, w) / \delta w_0 > \delta \Phi_1(w, w_0) / \delta w_0\).

(b) \(\Phi_1\) increases with the buyer’s confidence level \(\beta_s\).

**Proof.** In Appendix.

As stated in Proposition 3(a), the supplier can increase the option price and option exercise price to hedge risk. Additionally, the promotion of option price contributes more to the supplier’s risk reduction than the increase of option exercise price. The explanation is similar to that of Corollary 1(d) and Proposition 1. Notably, compared with the result in Proposition 1, the influences of \((w_0, w_0)\) on the buyer and supplier are opposite. The supplier’s risk hedging behavior will bring higher risk to the buyer. However, the supplier cannot reduce risk by increasing price infinitely, and \((w_0, w_0)\) should satisfy the constraints of Assumptions (b), (c) and (d). This guarantees that the buyer is willing to buy option order. Proposition 3(b) indicates that the increase of the buyer’s risk aversion level will bring the supplier more loss.

The opposite effects of option price and option exercise price on the buyer and supplier are also found by Zhuo et al. (2018), and they report that “as the option price increases, the supplier’s expected profit increases, whereas the retailer’s expected profit decreases. As the exercise price increases, the supplier’s expected profit and risk increase, whereas the retailer’s expected profit and risk decrease”. With Propositions 1 and 3, this research further compares the effects of the option price and the option exercise price on the supplier and buyer’s CVaR, and shows that the increase of option price will benefit the supplier but cause more loss to the buyer than that caused by increasing the same amount of option exercise price. Therefore, if the supplier wants to attract less risk-averse buyers, she could offer a contract with a lower option price but a higher option exercise price.

In the Stackelberg game, the buyer and supplier consider their best strategies respectively. As a result, from Theorem 1, Propositions 1, 2 and 3, the supplier has the motivation to increase the option price and option exercise price in order to hedge her risk, which increases the buyer’s risk. Thus, to improve efficiency, we need to consider the channel coordination under option contract in order to enable the buyer and supplier to work together and achieve the minimum possible risk at the supply chain level.

5. Supply chain coordination

In this section, we investigate supply chain coordination under CVaR criterion. First, we present the centralized model, then analyze the coordination of the buyer-led supply chain.

5.1. Centralized supply chain

The supply chain’s risk is the risk from both the buyer and the supplier. Thus, we define the joint net loss CVaR of the supply chain as \(\Phi_c\), which is obtained by summing up the buyer’s net loss CVaR and the supplier’s net loss CVaR as \(\Phi_c \equiv \Phi_b + \Phi_s\). The optimal order quantity to minimize the centralized supply chain’s risk is to find the optimal solution to the following problem:

\[
\min_{Q, M} \Phi_c(Q, M) \tag{22}
\]

Its optimal solution \((Q, M)\) is showed in the following proposition.

**Proposition 4.** If \(\beta_b > \beta_s\), the optimal order quantities are

\[
\begin{cases}
Q = 0 \\
M = F^{-1}(1 - \beta_s)
\end{cases}
\tag{23}
\]

If \(\beta_b < \beta_s\), the optimal order quantities are

\[
\begin{cases}
Q = F^{-1}(1 - \beta_b) \\
M = F^{-1}(1 - \beta_b) - Q
\end{cases}
\tag{24}
\]

If \(\beta_b = \beta_s\), the optimal order quantities only need to satisfy \(Q + M = F^{-1}(1 - \beta_b)\).

**Proof.** In Appendix.

As stated in Proposition 4, all the optimal order quantities are only related with the decision makers’ risk preference. If the buyer has higher risk aversion compared with the supplier, more option order is preferred. This is because option provides flexibility to the buyer to hedge or transfer her risk. If the buyer is less risk-averse compared with the supplier, committed order is preferred and the risks at both sides need to be balanced.

In the following corollary, the order quantities to minimize the buyer’s CVaR (11) and the supply chain’s CVaR (23) and (24) are compared.

**Corollary 2.** If \(\beta_b > \beta_s\), then \(Q_b < Q, M_b > M_b\) and \(Q_b + M_b > Q_b + M_b\);

If \(\beta_b < \beta_s\), then \(Q_b + M_b = Q_b + M_b\).

**Proof.** These results are straightforward by Proposition 4, Theorem 1(c) and the relationship of \(\beta_b\) and \(\beta_s\).

The buyer with higher risk aversion tends to be more sensitive to market uncertainty, which leads her to prefer more options. At the supply chain level, more options and less committed order is a good choice in this case. On the contrary, if the risk aversion of the buyer is not higher than that of the supplier, intuitively it is better to let the buyer take more risk at the supply chain level, thus it does not differ much from optimizing the buyer’s own risk.

This can be observed from the second part of Corollary 2.

5.2. Decentralized supply chain and coordination

In the decentralized buyer-led supply chain, the buyer’s optimal order quantities \((Q_b, M_b)\) are decided first. We define the joint net loss CVaR of the decentralized supply chain as \(\Phi_d \equiv \Phi_b + \Phi_s\) with the buyer’s purchasing decisions \((Q_b, M_b)\), and obtain \(\Phi_d(w_b, w_s)\) with the following optimization problem:

\[
\min_{w_b \geq w_s \geq 0} \Phi_d(w_b, w_s) \tag{25}
\]

Its optimal solution is presented in Theorem 2.

**Theorem 2.**

(a) If \(\beta_b < \beta_s\), the supply chain’s CVaR \(\Phi_d\) increases with option price \(w_s\); otherwise, it decreases with increasing option price \(w_s\);

(b) If \(\beta_b < \beta_s\), the changes of option price \(w_s\) and option exercise price \(w_s\) have no impact on the supply chain’s CVaR \(\Phi_d\);

(c) If \(\beta_b < \beta_s\), then \(\delta \Phi_d(w_b, w_s) / \delta w_s < \delta \Phi_d(w_b, w_s) / \delta w_b\); otherwise, \(\delta \Phi_d(w_b, w_s) / \delta w_b \geq \delta \Phi_d(w_b, w_s) / \delta w_s\).

**Proof.** In Appendix.

The influence from option price relates closely with the decision maker’s risk preference. Theorem 2 reveals that how option price transfers risk. Part (a) of Theorem 2 can be deduced from Propositions 1 and 3. In the two propositions, the increase of option price benefits the supplier but hurts the buyer. In addition, if the buyer’s risk aversion is higher, when option price increases, the buyer’s risk increases more than the decrease of the supplier’s risk, thus the supply chain’s CVaR will increase; otherwise it will decrease. It implies that in such a buyer-led supply chain, the supplier cannot control the supply chain’s risk by adjusting the pricing of option contracts. Thus, the risk of the entire supply chain will be determined by the buyer’s order quantities. Therefore, in order to retain her own initiative in risk control, the supplier will prefer to choose a buyer with a different risk aversion level.

Analytical results for the option exercise price are not obtained and its influence is investigated by the numerical studies in Section 6.

When the two parties have the same risk preference, the change in price (option price or option exercise price) will have the same effect on the change
in their risks. It means that if the supplier’s risk is increased (decreased) by a certain amount, the buyer’s risk will also be decreased (increased) by the same amount.

**Corollary 3.** $\partial \Phi_w(w_o, w_c)/\partial w_o$ is a decreasing function of the supplier’s confidence level $\beta_s$.

**Proof.** In Appendix.

For the change of option price, this corollary shows that the supplier’s risk preference has a direct influence on total risk. According to Theorem 2(a), if $\beta_b < \beta_s$, then $\partial \Phi_w(w_o, w_c)/\partial w_o > 0$. The value of $\partial \Phi_w(w_o, w_c)/\partial w_o$ decreases as $\beta_b$ increases and drops to zero until $\beta_b = \beta_s$, as indicated by Theorem 2(b). If $\beta_b > \beta_s$, then $\partial \Phi_w(w_o, w_c)/\partial w_o < 0$, and the value of $\partial \Phi_w(w_o, w_c)/\partial w_o$ decreases as $\beta_b$ increases. It indicates that when the supplier has a high risk aversion level (high confidence level $\beta_s$), the higher option price can alleviate the marginal risk of the whole supply chain. It is intuitive since for a very risk-averse supplier, she may not be willing to offer options to finance the buyer, because such a contract will transfer market uncertainties to herself, but a high option price can guarantee much of her benefits at the beginning.

We then present the coordination conditions in the following theorem.

**Theorem 3.** If the parameters satisfy

$$\beta_b = \beta_s \quad (26)$$

or

$$\beta_b < \beta_s \quad \begin{cases} 
\beta_b < \beta_s, \\
(\beta_b - \beta_s)w_o + (1 - \beta_s)w_o = (1 - \beta_b)w
\end{cases} \quad (27)$$

then the option contracts coordinate the supply chain under CVaR criterion.

**Proof.** In Appendix.

As stated in Theorem 3, there are two approaches to coordinate the supply chain. When the buyer and supplier have the same risk preference, the supply chain is already coordinated. This result is easy to understand by Theorem 2(b), where the risk shifting between the two parties will not affect the total supply chain risk.

The other coordination mechanism is a linear requirement expressed by (27). When the supplier is more risk-averse, the supply chain can be coordinated, where the option price and option exercise price should satisfy the specified linear equation which also includes the product wholesale price and both sides’ confidence levels.

With the risk measure of mean-variance, Zhuo et al. (2018) find that by leveraging the exercise price, channel coordination can be achieved when the buyer’s risk aversion threshold falls within certain intervals. This study adopts the CVaR measure and demonstrates that a supply chain can be coordinated with option contract only if the supplier’s risk preference is not less than that of the buyer. This finding contributes to the literature by incorporating both the buyer and supplier perspectives.

We reveal that when the buyer holds a higher risk aversion level ($\beta_b > \beta_s$), the supply chain cannot be coordinated. Choi et al. (2008) also point out that in some cases supply chain coordination is not available owing to the problem of “matching” the risk preference of the individual supply chain agents. Furthermore, Gan et al. (2005) find that the buy-back contract cannot coordinate the supply chain when the natural downside-risk faced by the retailer is higher than the level that the retailer would accept. Similarly, our results show that the mismatching of the risk preference of the supply chain members results in the failure of supply chain coordination. This is also consistent with the result in Corollary 2, where the centralized model’s optimal total order quantity of options order and committed order are larger than that of the decentralized model, which makes the coordination impossible.

6. **Numerical study**

The effects of option price, option exercise price and channel coordination are investigated by numerical examples in this section.

6.1. **The effects of option price and option exercise price**

As mentioned in Propositions 1 and 3, we have analytically characterized how the buyer’s and supplier’s net loss CVaR change with option price and option exercise price. However, the analysis at the supply chain level is intricate. Theorem 2 presents the relationship between the supply chain’s joint net loss CVaR and the option price but the mechanism in option exercise price is not clarified yet. Here, we investigate the effect of option exercise price at the supply chain level by numerical study.

Assume that the stochastic market demand follows an exponential distribution $\Phi(0.6)$. The final market price, wholesale price and production cost are $p = 13$, $w = 5$ and $c = 3$, respectively. $w_o$ keeps unchanged as $w_o = 1$ in Figs. 1 and 3, and $w_c$ remains unchanged as $w_c = 6$ in Figs. 2 and 4.

Figs. 1 and 2 show the influence of $w_o$ and $w_c$ on the decentralized supply chain joint CVaR, $\Phi_w$. Theorem 2 has revealed that $\Phi_w$ is affected by not only the option-related price but also the decision maker’s risk preference. Thus, we consider three cases of $\beta_b = \beta_s$, $\beta_b < \beta_s$ and $\beta_b > \beta_s$ ($\beta_b, \beta_s$ are set as $(0.7, 0.7)$, $(0.7, 0.75)$ and $(0.7, 0.65)$, respectively. The trend showed in Fig. 1 is consistent with the analytical result in
Obviously, from the numerical example, the change of option exercise price and production cost as sub-section 6.1. Considering the risk preference of (27) that $\beta_b > \beta_s$, we set $(\beta_b, \beta_s) = (0.6, 0.7)$ in Table 2 and $(\beta_b, \beta_s) = (0.6, 0.8)$ in Table 3, respectively. As for the linear combination of $(\beta_b, \beta_s)$, we have $\Phi_b = \Phi_s + \Phi_b$, but do not minimize the supply chain risk. Under the corresponding $(\beta_b, \beta_s)$ and $(w_0, w_1)$, the supply chain’s net loss CVaR $\Phi_t$, the buyer’s net loss CVaR $\Phi_b$, and the supplier’s net loss CVaR $\Phi_s$ are calculated and presented in the following two tables.

As for the situation $\beta_b > \beta_s$, where the supply chain cannot be coordinated, we calculate the buyer’s and the supply chain’s optimal order quantities to show the impossibility of coordination. In this case, we assume the option price as $w_1 = 1$ and option exercise price as $w_0 = 6$, and different combinations of $(\beta_b, \beta_s)$ are set to satisfy $\beta_b > \beta_s$. The buyer’s optimal order quantities $(Q_b, M_b)$ and the supply chain’s optimal order quantities $(Q_s, M_s)$ are calculated under different confidence level combinations $(\beta_b, \beta_s)$ in Table 4. Obviously, there are $Q_s < Q_b$ and $M_s > M_b$, which are consistent with Corollary 2.

### 7. Conclusions

Option contracts are used by many firms, especially small firms, to tackle difficulties in financing their working capital. This paper studies a one-buyer-one-supplier and buyer-led supply chain with option contract incorporating the supplier and the buyer’s risk preference under CVaR risk measure. Some interesting results are found, which enrich our understanding of option contract design. Firstly, as the option writer, the supplier would prefer to increase option price for risk hedging, while the buyer has to bear more risk. Secondly, the influence on supply chain risk depends on the risk preference of the buyer and supplier. When they have the same risk preference, pricing of options has no influence on total risk. If the buyer has lower risk aversion, the increase of option price will reduce total risk. Otherwise, with a more risk-averse buyer, the supply chain’s risk will increase. Thirdly, we prove that the supply chain can be coordinated with option contract.
under CVaR minimization. Two coordination approaches are presented. One requires that the buyer and supplier have the same risk preference. The other is a specified linear equation with option price, option exercise price, wholesale price, and decision makers’ confidence levels. We also find if the buyer is more risk averse, the supply chain cannot be coordinated.

Our results demonstrate the effects of option contract on the risk at the buyer and the supplier, which provide guides for contract design. Further, the results also suggest that the good strategy for a risk averse supplier is to work with a less risk averse buyer, which will reduce the total supply chain risk and also the risk at the supplier’s side.

This research has some limitations. One is that this study only considers CVaR minimization without considering the expected profits for the buyer and supplier. Clearly it will be a much more complex research problem if the expected profits are considered. Second, for a more risk averse buyer, the supply chain risk coordination is impossible according to our results. Nevertheless, for a highly risk averse buyer, multiple sourcing is a good strategy for risk management. Yet this study is limited to a single supplier. Thus, our future work will investigate mean-CVaR supply chain risk management and whether supply chain risk coordination is possible for a highly risk averse buyer using a multiple sourcing strategy.

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Appendix

Proof of Lemma 1. For (a), based on (1) and (2) as well as the definition of net loss, we directly have

\[ L_{\alpha} \geq -pQ + wQ + w_0M \]

and

\[-pQ + wQ + w_0M \geq L_{\alpha} \geq -p(Q + M) + w_0M + wQ + w_0M \]

Therefore, \[ L_{\alpha_1} \geq L_{\alpha_2} \geq L_{\alpha_3} = -p(Q + M) + w_0M + wQ + w_0M \] and this proves part (a). Similarly, \[ L_{\alpha_1} \geq L_{\alpha_2} \geq L_{\alpha_3} \] can be proved. This completes this Proof.

Proof of Theorem 1. From the definition of net loss (3) and auxiliary function (5), we have

\[ R_0(Q, M, \alpha) = \alpha + \frac{1}{1 - \bar{\rho}_b} \int_{0}^{+\infty} [L_1(Q, M, x) - \alpha]^+ f(x) dx = \alpha + \frac{1}{1 - \bar{\rho}_b} \int_{0}^{Q} [L_1 - \alpha]^+ f(x) dx + \int_{Q}^{Q+M} [L_2 - \alpha]^+ f(x) dx + \int_{Q+M}^{+\infty} [L_3 - \alpha]^+ f(x) dx \]

(A.1)

We first analyze \( \alpha_0 \). Based on Lemma 1, here we discuss the following cases:

Case 1. \( \alpha \geq w_0M + wQ \).
\( \alpha \geq w_0M + wQ \) means \( L_0(Q, M, x) - \alpha \leq 0 \). Thus, by (A.1)

\[ R_0(Q, M, \alpha) \equiv R_{\alpha_0}(Q, M, \alpha) = \alpha \]

Obviously, \( R_{\alpha_0}(Q, M, \alpha) \) increases with \( \alpha \).

Case 2. \( w_0M + wQ \geq -pQ + wQ + w_0M \).
In this case, \( [L_{\alpha_2} - \alpha]^+ = 0 \) and \( [L_{\alpha_3} - \alpha]^+ = 0 \). When \( L_0(x) - \alpha > 0 \), we have

\[ x < \frac{w_0M + wQ - \alpha}{p} \]

and from \( w_0M + wQ > \alpha \geq -pQ + wQ + w_0M \),

\[ 0 < x \leq \frac{w_0M + wQ - \alpha}{p} < Q \]

(A.2)

Thus,

\[ R_0(Q, M, \alpha) \equiv R_{\alpha_0}(Q, M, \alpha) = \alpha + \frac{1}{1 - \bar{\rho}_b} \int_{0}^{w_0M + wQ - \alpha} [L_1 - \alpha] f(x) dx \]

and

\[ \frac{\partial R_{\alpha_0}(Q, M, \alpha)}{\partial \alpha} = 1 - \frac{1}{1 - \bar{\rho}_b} f \left( \frac{w_0M + wQ - \alpha}{p} \right) \]

\[ \frac{\partial^2 R_{\alpha_0}(Q, M, \alpha)}{\partial \alpha^2} = \frac{1}{p(1 - \bar{\rho}_b)} f \left( \frac{w_0M + wQ - \alpha}{p} \right) \geq 0 \]

Therefore, \( R_{\alpha_0}(Q, M, \alpha) \) is a convex function of \( \alpha \) and the optimal \( \alpha_0 \) can be calculated when \( \partial R_{\alpha_0}(Q, M, \alpha)/\partial \alpha = 0 \), which is

\[ \alpha_{\alpha_0} = wQ + w_0M - pF^{-1}(1 - \bar{\rho}_b) \]

Additionally, for \( \alpha_{\alpha_0} \), by (A.2), it should be subject to the constraint of

\[ Q > F^{-1}(1 - \bar{\rho}_b) \]
With (A.3), we have
\[ R_2(Q, M, a_0) = wQ + w_0M - pF^{-1}(1 - \beta_0) + \frac{p}{1 - \beta_0} \int_0^1 F(x)dx \] (A.5)
and
\[ \frac{\partial R_2(Q, M, a_0)}{\partial Q} = w > 0 \]
\[ \frac{\partial R_2(Q, M, a_0)}{\partial M} = w_0 > 0 \]

Therefore, with the constraint (A.4), we obtain the optimal order quantities as
\[ Q_{b_2} = F^{-1}(1 - \beta_0) \] and
\[ M_{b_2} = 0 \] and there is

**Case 3.** \( -pQ + wQ + w_0M > \alpha > -p(Q + M) + w_0M + wQ + w_0M \)

In this case, \( [L_{b_2} - \alpha] > 0 \). When \( L_{b_2}(x) - \alpha > 0 \), we have
\[ x < \frac{w_0M + wQ - w_0Q - \alpha}{p - w_0} \]
and from \( -pQ + wQ + w_0M \geq \alpha \geq -p(Q + M) + w_0M + wQ + w_0M \),
\[ Q < \frac{w_0M + wQ - w_0Q - \alpha}{p - w_0} < Q + M \] (A.6)

Thus,
\[ R_0(Q, M, \alpha) = R_0(Q, M, \alpha) = \alpha + \frac{1}{1 - \beta_0} \int_0^1 [L_{b_1} - \alpha]f(x)dx + \int_0^1 [L_{b_2} - \alpha]f(x)dx \]
and
\[ \frac{\partial R_{b_1}(Q, M, \alpha)}{\partial \alpha} = 1 - \frac{1}{1 - \beta_0} F\left(\frac{w_0M + wQ - w_0Q - \alpha}{p - w_0}\right) \]
\[ \frac{\partial^2 R_{b_1}(Q, M, \alpha)}{\partial \alpha^2} = 1 - \frac{1}{1 - \beta_0} F\left(\frac{w_0M + wQ - w_0Q - \alpha}{p - w_0}\right) \geq 0 \]

Similarly, the optimal \( \alpha \) can be calculated when \( \frac{\partial R_{b_1}(Q, M, \alpha)}{\partial \alpha} = 0 \), which is
\[ a_{b_3} = w_0M + wQ - w_0Q - (p - w_0)F^{-1}(1 - \beta_0) \] (A.7)

Additionally, for \( a_{b_3} \), by (A.6), it should be subject to the constraint of \( Q + M > F^{-1}(1 - \beta_0) > Q \). With (A.7), we have
\[ R_{b_1}(Q, M, a_{b_3}) = wQ + w_0M - w_0Q - (p - w_0)F^{-1}(1 - \beta_0) + \frac{p - w_0}{1 - \beta_0} \int_0^1 F(x)dx + \frac{p}{1 - \beta_0} \int_0^1 F(x)dx \]
and
\[ \frac{\partial R_{b_1}(Q, M)}{\partial Q} = w - w_0 + \frac{w_0F(Q)}{1 - \beta_0} \]
\[ \frac{\partial R_{b_1}(Q, M)}{\partial M} = w_0 > 0 \]

Obviously, \( \frac{\partial R_{b_1}(Q, M)}{\partial Q} \) increases with \( Q \) and we have the constraint \( Q + M > F^{-1}(1 - \beta_0) > Q \). Let \( \frac{\partial R_{b_1}(Q, M)}{\partial Q} = \frac{\partial R_{b_1}(Q, M)}{\partial M} \), we have
\[ Q_{b_1} = F^{-1}(w_0 + w_0 - 1 - \beta_0)/w_0 \]. When \( Q < Q_{b_1} \), it is obvious that \( \frac{\partial R_{b_1}(Q, M)}{\partial Q} < \frac{\partial R_{b_1}(Q, M)}{\partial M} \), thus to minimize \( R_{b_1}(Q, M) \), increasing \( Q \) is better than increasing \( M \). When \( Q \geq Q_{b_1} \), then \( \frac{\partial R_{b_1}(Q, M)}{\partial Q} \geq \frac{\partial R_{b_1}(Q, M)}{\partial M} \), thus increasing \( M \) is better than increasing \( Q \). Therefore, the optimal order quantities are
\[ Q_{b_3} = F^{-1}(w_0 + w_0 - 1 - \beta_0)/w_0 \] and
\[ M_{b_3} = F^{-1}(1 - \beta_0) - Q_{b_3} \]. By Assumption (d), it is obvious that \( Q_{b_3} < F^{-1}(1 - \beta_0) \), thus
\[ M_{b_3} > 0 \].

**Case 4.** \( \alpha \leq -p(Q + M) + w_0M + wQ + w_0M \)

In this case, we also have
\[ \alpha < (p - w_0)(Q + M) + wQ + w_0M \]
and then there are
\[ Q < \frac{w_0M + wQ - \alpha}{p} \]
and
\[
Q + M < \frac{w_M + wQ - \alpha}{p - w_o}
\]

\[
R_\alpha(Q, M, \alpha) \equiv R_{a4}(Q, M, \alpha) = + \int_0^Q [I_{\alpha3} - \alpha]f(x)dx + \int_Q^{Q+M} [I_{\alpha2} - \alpha]f(x)dx + \int_{Q+M}^{+\infty} [I_{\alpha1} - \alpha]f(x)dx
\]

and

\[
\frac{\partial R_{a4}(Q, M, \alpha)}{\partial \alpha} = \frac{-\beta_2}{1 - \beta_2} \leq 0
\]

Obviously, \(R_{a4}(Q, M, \alpha)\) decreases with increasing \(\alpha\).

Based on the above four cases, we need to find out in which case \(R_{a4}(Q, M, \alpha)\) obtains the minimum. Since

\[
\lim_{a \to w_M + wQ} R_{a2}(Q, M, \alpha) = R_{a2}(Q, M, w_M + wQ)
\]

\[
\lim_{a \to -pQ + wQ + w_M} R_{a3}(Q, M, \alpha) = R_{a3}(Q, M, -pQ + wQ + w_M)
\]

\[
\lim_{a \to -p(Q + M) + w_M + wQ + w_M} R_{a4}(Q, M, \alpha) = R_{a3}(Q, M, -p(Q + M) + w_M + wQ + w_M)
\]

thus \(R_{a4}(Q, M, \alpha)\) is a continuous function of \(\alpha\). Meanwhile, \(R_{a4}(Q, M, \alpha)\) decreases with \(\alpha\), \(R_{a4}(Q, M, \alpha)\) and \(R_{a4}(Q, M, \alpha)\) are convex functions of \(\alpha\) and \(R_{a4}(Q, M, \alpha)\) decreases with \(\alpha\). Therefore, the optimal \(\alpha_b\) can only come from Case 2 or Case 3. Notably, as

\[
\alpha = w_M + wQ - w_1 - (p - w_2)F^{-1}(1 - \beta_2)
\]

By Lemma 2 (c), the buyer’s CVaR denoted by \(\Phi_{w}\) is

\[
\Phi_{w}(Q, M, \alpha) = wQ + w_M - w_1 - (p - w_2)F^{-1}(1 - \beta_2) + \frac{p}{1 - \beta_2} \int_0^Q F(x)dx + \frac{p - w_2}{1 - \beta_2} \int_Q^Q F(x)dx
\]

The optimal order quantities are \((Q_{a3}, M_{a3})\) presented in Case 3. This completes this Proof.

**Proof of Corollary 1.** For part (a),

\[
\frac{\partial \Phi_{w}}{\partial w_o} = f^{-1}(1 - \beta_2)(w_2 + w_2 - w) \frac{1 - \beta_2}{w_2} \geq 0
\]

\[
\frac{\partial \Phi_{w}}{\partial w_e} = f^{-1}(1 - \beta_2)(w_2 + w_2 - w) \frac{1 - \beta_2}{w_2} \geq 0
\]

\[
\frac{\partial \Phi_{w}}{\partial w} = -f^{-1}(1 - \beta_2)(w_2 + w_2 - w) \frac{1 - \beta_2}{w_2} \leq 0
\]

Similarly, Part (b) is straightforward from part (a).

For part (c),

\[
\frac{\partial \Phi_{b}}{\partial \beta_0} = -f^{-1}(1 - \beta_2)(w_2 + w_2 - w) \frac{1 - \beta_2}{w_2} \leq 0
\]

\[
\frac{\partial \Phi_{b}}{\partial \beta_2} = -f^{-1}(1 - \beta_2) + f^{-1}(1 - \beta_2)(w_2 + w_2 - w) \frac{w_2 + w_2 - w}{w_2} \leq 0
\]

For part (d),

\[
\frac{\partial \Phi_{b}}{\partial \beta_0} = f^{-1}(1 - \beta_2)(w_2 + w_2 - w) \frac{1 - \beta_2}{w_2} \geq 0
\]

\[
\frac{\partial \Phi_{b}}{\partial \beta_2} = -f^{-1}(1 - \beta_2)(w_2 + w_2 - w) \frac{w_2 + w_2 - w}{w_2} \leq 0
\]

This completes the Proof.

**Proof of Proposition 1.** By introducing the buyer’s optimal order quantities (11) into the buyer’s net loss CVaR (10), we have \(\Phi_0(w_o, w_e)\) as

\[
\Phi_0(w_o, w_e) = wQ_o + w_Mo - w_1Q_o - (p - w_2)F^{-1}(1 - \beta_2) + \frac{p}{1 - \beta_2} \int_0^Q F(x)dx + \frac{p - w_2}{1 - \beta_2} \int_Q^Q F(x)dx
\]

The derivatives of \(\Phi_0(w_o, w_e)\) with respect to \(w_o\) and \(w_e\) are

\[
\frac{\partial \Phi_0}{\partial w_o} = M_o > 0
\]
and
\[
\frac{\partial \Phi_b(w_o, w_e)}{\partial w_o} = F^{-1}(1 - \beta_b) - Q_b - \frac{1}{1 - \beta_b} \int_{Q_b}^{\infty} F(x)dx
\]

Notably, there is \(1/(1 - \beta_b)\int_{Q_b}^{\infty} F(x)dx \leq F^{-1}(1 - \beta_b) - Q_b\), thus \(\partial \Phi_b(w_o, w_e)/\partial w_o > 0\). The difference of them is
\[
\frac{\partial \Phi_b(w_o, w_e)}{\partial w_o} - \frac{\partial \Phi_b(w_o, w_e)}{\partial w_e} = \frac{1}{1 - \beta_b} \int_{Q_b}^{\infty} F(x)dx \geq 0.
\]

This completes the Proof.

**Proof of Proposition 2.** From the definition of net loss (4) and auxiliary function (5), we have
\[
R_3(Q, M, \alpha) = \alpha + \frac{1}{1 - \beta} \int_0^{Q} \left[ \int_{Q}^{M} f(x)dx + \int_0^{Q} \left[ L_{12} - \alpha \right]^f(x)dx + \int_{Q}^{Q+M} \left[ L_{13} - \alpha \right]^f(x)dx \right](1 - \beta).
\]

For any committed order \(Q\) and option order \(M\), the optimal solution \(\alpha\) to minimize \(R_3(Q, M, \alpha)\) is decided by the following cases:

**Case 1.** \(\alpha > c(Q + M) - w_Q - w_o M\).

In this case, \(\alpha > L_2(Q, M, x)\), therefore
\[
R_3(Q, M, \alpha) = R_{13}(Q, M, \alpha) = \alpha
\]

Obviously, \(R_{13}(Q, M, \alpha)\) increases with \(\alpha\).

**Case 2.** \(c(Q + M) - w_Q - w_o M \geq \alpha > c(Q + M) - w_Q - w_o M\).

Clearly, \(\alpha \leq L_1\) and \(L_{13} - \alpha\) = 0. If \(L_{12} - \alpha \geq 0\), we have
\[
c(Q + M) - w_Q - w_o M + w_o Q - \alpha \geq x
\]

Based on the case condition, we have
\[
Q \leq c(Q + M) - w_Q - w_o M + w_o Q - \alpha < Q + M
\]
(A.11)

thus
\[
R_3(Q, M, \alpha) = R_{23}(Q, M, \alpha) = \alpha + \frac{1}{1 - \beta} \int_0^{Q} \left[ \int_{Q}^{M} f(x)dx + \int_0^{Q} \left[ L_{12} - \alpha \right]^f(x)dx \right]
\]

and
\[
\frac{\partial R_{23}(Q, M, \alpha)}{\partial \alpha} = 1 - \frac{1}{1 - \beta} f(c(Q + M) - w_Q - w_o M + w_o Q - \alpha)
\]

\[
\frac{\partial^2 R_{23}(Q, M, \alpha)}{\partial \alpha^2} = \frac{1}{(1 - \beta)w_o} f(c(Q + M) - w_Q - w_o M + w_o Q - \alpha) \geq 0
\]

Thus, the optimal \(\alpha\) can be computed by solving \(\partial R_{23}(Q, M, \alpha)/\partial \alpha = 0\) as
\[
\alpha = c(Q + M) - w_Q - w_o M + w_o Q - w_o F^{-1}(1 - \beta)
\]

Additionally, through (A.11), we have \(Q + M > F^{-1}(1 - \beta)\) > \(Q\). With the expression of \(\alpha\), we have
\[
R_{23}(Q, M) = c(Q + M) - w_Q - w_o M + w_o Q - w_o F^{-1}(1 - \beta) + \frac{w_o}{1 - \beta} \int_0^{Q} f(x)dx
\]

Similar to the discussion of the buyer in Theorem 1, we find the optimal order quantity to minimize \(R_{23}(Q, M)\) through investigating how changes of \(Q\) and \(M\) influence \(R_{23}(Q, M)\). The derivatives of \(R_{23}(Q, M)\) with respect to \(Q\) and \(M\) are
\[
\frac{\partial R_{23}(Q, M)}{\partial Q} = c - w + w_o - \frac{w_o F(Q)}{1 - \beta}
\]

and
\[
\frac{\partial R_{23}(Q, M)}{\partial M} = c - w_o
\]

Assume the optimal order quantities to minimize \(\Phi_b\) are \((Q_o, M_o)\). Notably, when \(c \leq w_o\), \(\partial R_{23}(Q, M, \alpha)/\partial M < 0\), which means that under this situation the increase of option order helps reduce the supplier’s net loss, thus \(Q_o\) will be infinite. As for the optimal committed order quantity, we consider the relationship between \(\partial R_{23}(Q, M, \alpha)/\partial M\) and \(\partial R_{23}(Q, M, \alpha)/\partial Q\). If \(\partial R_{23}(Q, M, \alpha)/\partial Q \geq \partial R_{23}(Q, M, \alpha)/\partial M\) or \(Q < F^{-1}(w_o + w - w_o (1 - \beta))/w_o\), we have \(Q_o = 0\). If \(\partial R_{23}(Q, M, \alpha)/\partial Q < \partial R_{23}(Q, M, \alpha)/\partial M\) or \(Q > F^{-1}(w_o + w - w_o (1 - \beta))/w_o\), the optimal committed order quantity can be \(Q = F^{-1}(1 - \beta)\). To find the optimal one, we let them into \(R_{23}(Q, M)\) and make a difference as
Thus, if \((c + w - w)/w > \int_0^1 F(x)dx/\{F^{-1}(1 - \beta_0)(1 - \beta_0)\}\), the optimal committed order quantity is \(Q_s = 0\). Otherwise, it is \(Q_s = F^{-1}(1 - \beta_0)\).

When \(c > w_0\), \(\partial R_s(Q, M)/\partial M > 0\) and then a comparison between \(\partial R_s(Q, M)/\partial M\) and \(\partial R_s(Q, M)/\partial Q\) is made to derive the buyer's optimal order quantities. If \(\partial R_s(Q, M)/\partial Q \geq \partial R_s(Q, M)/\partial M\) or \(Q \leq F^{-1}(w_0 + w - w)(1 - \beta_0)/w_0\), we have \(Q_s = 0\) and \(M_s = F^{-1}(1 - \beta_0)\). If \(\partial R_s(Q, M)/\partial Q < \partial R_s(Q, M)/\partial M\) or \(Q > F^{-1}(w_0 + w - w)(1 - \beta_0)/w_0\), the optimal order quantities can be \(Q_s = F^{-1}(1 - \beta_0)\) and \(M_s = 0\). To find the optimal one, we let them into \(R_s(Q, M)\) and make a difference as

\[
R_s(0, F^{-1}(1 - \beta_0)) - R_s(F^{-1}(1 - \beta_0), 0) = (w - w_0 - w)F^{-1}(1 - \beta_0) + \frac{w_0}{1 - \beta_0} \int_0^1 F(x)dx
\]

Thus, if \((w_0 + w - w)/w > \int_0^1 F(x)dx/\{F^{-1}(1 - \beta_0)(1 - \beta_0)\}\), the optimal order quantities are \(Q_s = 0\) and \(M_s = F^{-1}(1 - \beta_0)\). Otherwise, they are \(Q_s = F^{-1}(1 - \beta_0)\) and \(M_s = 0\).

**Case 3.** \(\alpha \leq c(Q + M) - wQ - wM - M\).

In this case, with \(\alpha \leq \beta_3\), there are

\[
R_s(Q, M, \alpha) = R_s(Q, M, \alpha) = \alpha + \frac{1}{1 - \beta_0} [\int_0^Q (M_3 - \alpha)f(x)dx + \int_0^{Q + M} (M_3 - \alpha)f(x)dx + \int_0^{Q + M} (M_3 - \alpha)f(x)dx]
\]

and

\[
\frac{\partial R_s(Q, M, \alpha)}{\partial \alpha} = -\frac{\beta_0}{1 - \beta_0} \leq 0
\]

Obviously, \(R_s(Q, M, \alpha)\) decreases with \(\alpha\).

Similar to the analysis in **Theorem 1**, it is also obvious that \(R_s(Q, M, \alpha)\) is a continuous function of \(\alpha\), and \(R_s(Q, M, \alpha)\) increases with \(\alpha\), \(R_s(Q, M, \alpha)\) is a convex function of \(\alpha\) and \(R_s(Q, M, \alpha)\) decreases with increasing \(\alpha\). Therefore, the supplier's optimal threshold of loss is obtained from \(R_s(Q, M, \alpha)\) as

\[
\alpha_0 = c(Q + M) - wQ - wM + Q - wF^{-1}(1 - \beta_0)
\]

and

\[
\Phi_s(Q, M) = c(Q + M) - wQ - wM + Q - wF^{-1}(1 - \beta_0)
\]

with the optimal order quantity to minimize it as \((Q_s, M_s)\) described in Case 2. This completes the Proof.

**Proof of Proposition 3.** By introducing the buyer's optimal order quantities (11) into the supplier's net loss CVaR (16), we have \(\Phi_s(w_0, w_1)\) as

\[
\Phi_s(w_0, w_1) = c(Q_0 + M_0) - wQ_0 - w_0M_0 + Q_0 - w_0F^{-1}(1 - \beta_0) + \frac{w_0}{1 - \beta_0} \int_{Q_0}^{F^{-1}(1 - \beta_0)} F(x)dx
\]

and the derivatives of \(\Phi_s(w_0, w_1)\) with respect to \(w_0\) and \(w_1\) are

\[
\frac{\partial \Phi_s(w_0, w_1)}{\partial w_0} = -M_0 + \frac{F^{-1}(1 - \beta_0)}{1 - \beta_0} \frac{\partial Q_0}{\partial w_0}
\]

and

\[
\frac{\partial \Phi_s(w_0, w_1)}{\partial w_1} = Q_0 - F^{-1}(1 - \beta_0) + \frac{1}{1 - \beta_0} \int_{Q_0}^{F^{-1}(1 - \beta_0)} F(x)dx + \frac{F^{-1}(1 - \beta_0)}{1 - \beta_0} \frac{\partial Q_0}{\partial w_1}
\]

Notably, since \(\frac{1}{1 - \beta_0} \int_{Q_0}^{F^{-1}(1 - \beta_0)} F(x)dx \leq F^{-1}(1 - \beta_0) - Q_0\), thus with (A.8), (A.9) and the constraint of (21), we obtain \(\Phi_s(w_0, w_1)/\partial w_0 < 0\) and \(\Phi_s(w_0, w_1)/\partial w_1 < 0\). The difference is made as

\[
\frac{\partial \Phi_s(w_0, w_1)}{\partial w_0} - \frac{\partial \Phi_s(w_0, w_0)}{\partial w_0} = F^{-1}(1 - \beta_0) - F^{-1}(1 - \beta_0) + \frac{1}{1 - \beta_0} \int_{Q_0}^{F^{-1}(1 - \beta_0)} F(x)dx
\]

By the constraint of (21), obviously, \(\frac{\partial \Phi_s(w_0, w_1)}{\partial w_0} - \frac{\partial \Phi_s(w_0, w_1)}{\partial w_1} > 0\). For part (b), since \(\frac{\partial \Phi_s}{\partial w_1} \leq 0\), the derivative of \(\Phi_s(w_0, w_1)\) in \(\beta_0\) is

\[
\frac{\partial \Phi_s(w_0, w_1)}{\partial \beta_0} = \beta_0 + \beta_0 - 2 \frac{\partial Q_0}{\partial \beta_0} \geq 0
\]
This completes the Proof.

**Proof of Proposition 4.** In the Proof of Theorem 1 (In Appendix), there are two cases available for minimizing the buyer’s CVaR, while there is only one case for the supplier, which is showed in Proposition 2. Therefore, for the joint CVaR, we have the following two cases. The first case can be derived as:

\[
\Phi_i \equiv \Phi_b + \Phi_s = c(Q + M) - w_i F^{-1}(1 - \beta_i) - (p - w_s) F^{-1}(1 - \beta_s) + \frac{p}{1 - \beta_b} \int_0^{F^{-1}(1 - \beta_b)} F(x)dx + \frac{1}{1 - \beta_s} \int_Q^{F^{-1}(1 - \beta_s)} F(x)dx - \frac{1}{1 - \beta_b} \int_Q^{F^{-1}(1 - \beta_b)} F(x)dx
\]

where \(\Phi_b\) comes from (10) and \(\Phi_s\) comes from (16). Based on the decision maker’s constraints (9) and (15), the constraint of this case is

\[
Q + M > \max\{F^{-1}(1 - \beta_b), F^{-1}(1 - \beta_s)\} > \min\{F^{-1}(1 - \beta_b), F^{-1}(1 - \beta_s)\} > Q
\]

The other case is derived as

\[
\Phi_i \equiv \Phi_b + \Phi_s = c(Q + M) + w_i Q - w_i F^{-1}(1 - \beta_i) - p F^{-1}(1 - \beta_s) + \frac{p}{1 - \beta_b} \int_0^{F^{-1}(1 - \beta_b)} F(x)dx + \frac{w_s}{1 - \beta_s} \int_Q^{F^{-1}(1 - \beta_s)} F(x)dx
\]

where \(\Phi_b\) comes from (A.5) and \(\Phi_s\) comes from (16). Based on the decision maker’s constraints (A.4) and (15), the constraint of this case is

\[
Q + M > F^{-1}(1 - \beta_b) > Q > F^{-1}(1 - \beta_s)
\]

and that means \(\beta_b < \beta_s\).

According to the definition of the supply chain’s CVaR, we discuss the problem in the following two cases:

**Case 1.** Under constraint (A.13).

The derivatives of \(\Phi_i(Q, M)\) with respect to \(Q\) and \(M\) are

\[
\frac{\partial \Phi_i(Q, M)}{\partial Q} = c + w_i F(Q) \left(\frac{\beta_b - \beta_i}{1 - \beta_b} \right) > 0
\]

\[
\frac{\partial \Phi_i(Q, M)}{\partial M} = c > 0
\]

When \(\beta_b > \beta_i\), \(\partial \Phi_i(Q, M)/\partial Q > \partial \Phi_i(Q, M)/\partial M > 0\) and \(Q + M > F^{-1}(1 - \beta_i) > F^{-1}(1 - \beta_b) > Q\), thus

\[
\begin{cases}
Q = 0 \\
M = F^{-1}(1 - \beta_i)
\end{cases}
\]

When \(\beta_b < \beta_i\), \(\partial \Phi_i(Q, M)/\partial Q < \partial \Phi_i(Q, M)/\partial M\) and \(Q + M > F^{-1}(1 - \beta_i) > F^{-1}(1 - \beta_b) > Q\), thus

\[
\begin{cases}
Q = F^{-1}(1 - \beta_i) \\
M = F^{-1}(1 - \beta_b) - Q_c
\end{cases}
\]

When \(\beta_b = \beta_i\), under constraint \(Q + M > F^{-1}(1 - \beta_b) = F^{-1}(1 - \beta_i) > Q\), \((Q_c, M_c)\) only need to satisfy \(Q_b + M_b = F^{-1}(1 - \beta_i)\).

**Case 2.** Under constraint (A.15).

The derivatives of \(\Phi_i(Q, M)\) with respect to \(Q\) and \(M\) are

\[
\frac{\partial \Phi_i(Q, M)}{\partial Q} = c + w_i - \frac{w_s}{1 - \beta_i} F(Q)
\]

\[
\frac{\partial \Phi_i(Q, M)}{\partial M} = c
\]

Under constraint (15) of \(F^{-1}(1 - \beta_i) > Q\), \(\partial \Phi_i(Q, M)/\partial Q > \partial \Phi_i(Q, M)/\partial M > 0\). Hence, the optimal solution is

\[
\begin{cases}
Q = F^{-1}(1 - \beta_i) \\
M = F^{-1}(1 - \beta_b) - Q_c
\end{cases}
\]

Notably, there are two solutions for the situation \(\beta_b > \beta_i\); thus, we make a comparison of their corresponding CVaR to find the optimal one. Their difference is

\[
\Phi_i(0, F^{-1}(1 - \beta_i)) - \Phi_i(F^{-1}(1 - \beta_b), F^{-1}(1 - \beta_i) - F^{-1}(1 - \beta_s)) = -w_i \left[1 - \frac{1}{1 - \beta_b} \int_{F^{-1}(1 - \beta_b)}^{F^{-1}(1 - \beta_i)} F(x)dx + \frac{w_s}{1 - \beta_s} \int_Q^{F^{-1}(1 - \beta_s)} F(x)dx \right] < 0
\]

Thus for \(\beta_b > \beta_i\), the optimal order quantities are \(Q = 0\) and \(M = F^{-1}(1 - \beta_i)\), and the supply chain’s CVaR is characterized by (A.12). This completes the Proof.

**Proof of Theorem 2.** By introducing the buyer’s optimal order quantities (11) into the supply chain’s net loss CVaR (A.12), we have \(\Phi_i(w_b, w_i)\) as
\( \Phi_0(w_0, w_0) = c(Q_b + M_b) - w_0F^{-1}(1 - \beta_s) - (p - w_0)F^{-1}(1 - \beta_b) + \frac{p}{1 - \beta_b} \int_0^{F^{-1}(1 - \beta_b)} F(x)dx + w_0[\frac{1}{1 - \beta_s} \int_0^{F^{-1}(1 - \beta_s)} F(x)dx - \frac{1}{1 - \beta_b} \int_0^{F^{-1}(1 - \beta_b)} F(x)dx] \)  

(A.16)

and the derivatives of \( \Phi_0(w_0, w_0) \) with respect to \( w_0 \) and \( w_i \) are

\[
\frac{\partial \Phi_0}{\partial w_0} = (w_0 + w_i - w)(w_0 + w_i - w) \frac{\partial Q_b}{\partial w_0} - \frac{\partial Q_0}{\partial w_i} \frac{\partial Q_b}{\partial w_i} = (w_0 + w_i - w)(w_0 + w_i - w) \frac{\partial Q_b}{\partial w_0} - \frac{\partial Q_0}{\partial w_i} \frac{\partial Q_b}{\partial w_i}
\]

and 

\[
\frac{\partial \Phi_0}{\partial w_i} = F^{-1}(1 - \beta_b) - F^{-1}(1 - \beta_s) + (w_0 + w_i - w) \frac{\partial Q_b}{\partial w_i} - \frac{\partial Q_0}{\partial w_i} \frac{\partial Q_b}{\partial w_i} + [\frac{1}{1 - \beta_s} \int_0^{F^{-1}(1 - \beta_s)} F(x)dx - \frac{1}{1 - \beta_b} \int_0^{F^{-1}(1 - \beta_b)} F(x)dx]
\]

(A.17)

Clearly, if \( \beta_s = \beta_b \), then \( \frac{\partial \Phi_0}{\partial w_0} = 0 \) and \( \frac{\partial \Phi_0}{\partial w_i} = 0 \) with (A.8), if \( \beta_s < \beta_b \), then \( \frac{\partial \Phi_0}{\partial w_0} > 0 \), while if \( \beta_s > \beta_b \), then \( \frac{\partial \Phi_0}{\partial w_0} < 0 \). The difference between (A.18) and (A.17) is

\[
\frac{\partial \Phi_0}{\partial w_i} - \frac{\partial \Phi_0}{\partial w_i} = [F^{-1}(1 - \beta_b) - F^{-1}(1 - \beta_s)] + \frac{w_0 + w_i - w}{1 - \beta_b} - \frac{w_0 + w_i - w}{1 - \beta_s} + \frac{F^{-1}(1 - \beta_s) \int_0^{F^{-1}(1 - \beta_s)} F(x)dx - F^{-1}(1 - \beta_b) \int_0^{F^{-1}(1 - \beta_b)} F(x)dx}{1 - \beta_b} - \frac{F^{-1}(1 - \beta_b) \int_0^{F^{-1}(1 - \beta_b)} F(x)dx - F^{-1}(1 - \beta_s) \int_0^{F^{-1}(1 - \beta_s)} F(x)dx}{1 - \beta_s}
\]

(A.18)

Let \( U(y) = F^{-1}(y) - (1/y) \int_0^{F^{-1}(y)} F(x)dx \), and its derivative is 

\[
U'(y) = F^{-1}(y) + \frac{1}{y^2} \int_0^{F^{-1}(y)} F(x)dx - \frac{1}{y^2} F(F^{-1}(y)f^{-1}(y) = \frac{1}{y^2} \int_0^{F^{-1}(y)} F(x)dx > 0
\]

Thus, if \( \beta_s > \beta_b \), then we have 

\[
[F^{-1}(1 - \beta_b) - F^{-1}(1 - \beta_s)] - [F^{-1}(1 - \beta_s) - F^{-1}(1 - \beta_b)] = 0 < 0
\]

Thus \( \frac{\partial \Phi_0}{\partial w_0} - \frac{\partial \Phi_0}{\partial w_i} = 0 < 0 \), when \( \frac{\partial \Phi_0}{\partial w_i} - \frac{\partial \Phi_0}{\partial w_i} > 0 \). This completes the Proof.

Proof of Corollary 3. By (A.17) and (A.8), 

\[
\frac{\partial \Phi_0}{\partial w_i} = (w_0 + w_i - w) \frac{\partial Q_b}{\partial w_0} - \frac{\partial Q_0}{\partial w_i} \frac{\partial Q_b}{\partial w_i} = (w_0 + w_i - w) \frac{\partial Q_b}{\partial w_0} - \frac{\partial Q_0}{\partial w_i} \frac{\partial Q_b}{\partial w_i} - w_i \frac{F^{-1}(1 - \beta_b)(w_0 + w_i - w)}{w_i (1 - \beta_b)^2} < 0
\]

This completes the Proof.

Proof of Theorem 3. By the optimal order quantities in Theorem 1 and Proposition 4, if \( \beta_s > \beta_b \), let \( Q_c = Q_b \) and \( M_c = M_b \), we have:

\[
0 = F^{-1}\left[\frac{1 - \beta_b}{w_i}(w_0 + w_i - w)\right]
\]

\[
F^{-1}(1 - \beta_b) - F^{-1}(1 - \beta_b) = F^{-1}(1 - \beta_b) - F^{-1}\left[\frac{1 - \beta_b}{w_i}(w_0 + w_i - w)\right]
\]

Since \( \beta_s \neq \beta_b \), the preceding two equalities cannot hold, thus for this case the supply chain cannot be coordinated.

When \( \beta_s < \beta_b \), let \( Q_c = Q_b \) and \( M_c = M_b \), we have:

\[
F^{-1}(1 - \beta_b) = F^{-1}\left[\frac{1 - \beta_b}{w_i}(w_0 + w_i - w)\right],
\]

\[
F^{-1}(1 - \beta_b) - F^{-1}(1 - \beta_b) = F^{-1}(1 - \beta_b) - F^{-1}\left[\frac{1 - \beta_b}{w_i}(w_0 + w_i - w)\right]
\]

Obviously, this case coordinates at \((\beta_s - \beta_b)w_i + (1 - \beta)w_0 = (1 - \beta_b)w_i\).

When \( \beta_s = \beta_b \), let \( Q_c = Q_b = Q_b + M_b \),

\[
F^{-1}(1 - \beta_b) = F^{-1}(1 - \beta_b)
\]

and the channel is coordinated. This completes the Proof.

References


